GPS, Geodesy, and the Ghost in the Machine

A Workshop for Surveyors and GIS Professionals

Michael L. Dennis, RLS, PE

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Michael L. Dennis
Geodetic Analysis, LLC
mld@geodeticanalysis.com

WORKSHOP ABSTRACT

Coordinates derived from GPS equipment are determined using complex algorithms that are often hidden within proprietary software — the "ghost in the machine". Users unfamiliar with the computational process can unwittingly generate positional errors ranging from a fraction of a foot to many miles. This problem persists despite efforts of vendors to streamline and simplify the GPS positioning process.

This workshop seeks to shed light on the GPS "black box" by 1) Explaining the main geodetic principles and terminology behind GPS; 2) Reducing blind reliance on GPS and GIS software; and 3) Providing practical information and tools for the GPS user. Topics include geodetic and vertical datums, map projections, "ground" coordinate systems, the geoid, NGS Datasheets and OPUS, GIS data compatibility, and an overview of (draft) APLS standards for spatial data accuracy and georeferencing. Numerous examples of positioning errors will be used to illustrate the peril of neglecting geodetic principles in modern surveying and mapping applications. A workbook will be provided that includes step-by-step GPS and geodetic computations. So bring your questions and your favorite everyday tools (calculator, laptop computer, data collector), and together we will purge the ghosts from your machines!

ACKNOWLEDGEMENTS

The creation of this workbook would not have been possible without the support and inspiration provided by Gabriel Bey and Rick Bunger. Special thanks also go to Dave Minkel, the National Geodetic Survey Arizona Geodetic Advisor, and to all members of the Geospatial Committee of the Arizona Professional Land Surveyors Association.

Today, GPS has thrust surveyors into the thick of geodesy which is no longer the exclusive realm of distant experts. Thankfully, in the age of microcomputers, the computational drudgery can be handled with software packages. Nevertheless, it is unwise to venture into GPS believing that knowledge of the basics of geodesy is, therefore, unnecessary. It is true that GPS would be impossible without computers, but blind reliance on the data they generate eventually leads to disaster.

Jan Van Sickle (2001, p. 126)

Note: This workbook is intended to accompany a presentation. Therefore some of the material may appear incomplete or be unclear if it is used without attending the presentation.

TABLE OF CONTENTS

Section **Page**

LIST OF EQUATIONS

LIST OF TABLES

LIST OF SYMBOLS

a Semi-major axis of geodetic ellipsoid; *also* semi-major axis of error ellipse

A Astronomic azimuth

b Semi-minor axis of geodetic ellipsoid, $b = a\left(1 - f\right)$; *also* semi-minor axis of error ellipse

c_X^n Value used to scale an n-dimensional standard error to a confidence level of X%

C Geopotential number

C_C Horizontal curvature correction factor (multiplied with straight horizontal distance)

C_{CR} Vertical correction for curvature and refraction (added to vertical distance)

CEP Circular Error Probable

DC_{AB} Dynamic correction applied to leveled height difference from point A to B

D_{grnd} Horizontal ground distance (parallel to ellipsoid)

D_S Slope distance

e First eccentricity of geodetic ellipsoid, $e = \sqrt{e^2} = \sqrt{2f - f^2}$

e^2 First eccentricity squared of geodetic ellipsoid, $e^2 = 2f - f^2$

E Easting coordinate (in the x-direction)

E_{95}^{N} Error (accuracy) north component at accuracy at 95% confidence

E_{95}^{E} Error (accuracy) east component at accuracy at 95% confidence

E_{95}^{U} Error (accuracy) up component at accuracy at 95% confidence

E_0 False easting (on central meridian) of map projection definition

f Geometric flattening of geodetic ellipsoid

g Gravity at the topographic surface of the Earth

$\bar{g}$ Mean gravity on the plumbline between the topographic surface and the geoid

h Ellipsoid height

H Orthometric height ("elevation")

H^D Dynamic height

i Instrument or GPS base station antenna height above station

K Constant for computing Helmert mean gravity, $K = 2{,}358{,}000\ \text{s}^2 = 1 / (4.24 \times 10^{-7}\ \text{s}^{-2}) = 1 / (\tfrac{1}{2} F - 2\pi G\rho)$, where F is the vertical gradient of gravity, G is the universal gravitational constant and ρ is the topographic density (assumed constant 2670 kg/m^3)

k Conformal map projection grid scale factor

k_o Grid scale factor on the central meridian for the Transverse Mercator projection (or on the central parallel for the Lambert Conformal Conic projection)

L Laplace correction

N Northing coordinate (in the y-direction)

N_0 False northing (where central meridian crosses latitude of grid origin) of map projection

N_G Geoid height = geoid separation = geoid undulation

Δn_{AB} Leveled height difference observed from point A to B

OC_{AB} Orthometric correction applied to leveled height difference from point A to B

r Prism rod or rover antenna height above station

R_G Geometric mean radius of curvature of geodetic ellipsoid

R_M Radius of curvature in the meridian of geodetic ellipsoid

R_N Radius of curvature in the prime vertical of geodetic ellipsoid

R_α Radius of curvature of geodetic ellipsoid in a specific azimuth, α

s Geodesic distance ("horizontal" distance on the ellipsoid)

SEP Spherical Error Probable

t Grid azimuth

$(t - T)$ Arc-to-chord ("second term") correction for converting grid to geodetic azimuths

X Earth-Centered, Earth-Fixed Cartesian coordinate in the X-direction (in equatorial plane and passing through Prime Meridian, i.e., 0° longitude)

Y Earth-Centered, Earth-Fixed Cartesian coordinate in the Y-direction (in equatorial plane and perpendicular to X-axis, i.e., passing through 90°E longitude)

Z Earth-Centered, Earth-Fixed Cartesian coordinate in the Z-direction (parallel to Earth's conventional spin axis and perpendicular to equatorial plane)

α_{AB} Geodetic azimuth from point A to point B

$\tilde{\alpha}_{AB}$ Approximate geodetic azimuth from point A to point B

γ Convergence angle

γ_0 Normal gravity on the GRS 80 ellipsoid at 45° latitude

δ Map projection distortion

Δ Denotes discrete change in a quantity, usually as final value minus initial value (e.g., for change in northing coordinate, $\Delta N = N_2 - N_1$)

$\Delta\lambda''$ Change in longitude in arc-seconds

$\Delta\varphi''$ Change in latitude in arc-seconds

ζ Geodetic zenith angle

η East-west component of the deflection of the vertical (in the prime vertical plane)

θ Horizontal error ellipse rotation angle

λ Geodetic longitude

λ_0 Longitude of central meridian for map projection

ν Zenith angle

ξ North-south component of the deflection of the vertical (in the meridian plane)

π Irrational number pi (ratio of circle circumference to diameter)

ρ Horizontal correlation

σ_E Standard error (east component)

σ_N Standard error (north component)

σ_{NE} Horizontal covariance

φ Geodetic latitude (on ellipsoid or sphere)

φ_0 Latitude of grid origin for map projection; central parallel for conical map projection

φ_{N} Latitude of north standard parallel for conical map projection

φ_{S} Latitude of south standard parallel for conical map projection

ψ Angle between two points on a sphere with vertex at center of sphere

TABLE OF USEFUL NUMERICAL VALUES

Symbol	Description	Numerical values
a	GRS-80 ellipsoid semi-major axis (identical to WGS-84 value)	6,378,137 m (*exact*) = 20,925,646.325 459 ift = 20,925,604.474 167 sft
f	GRS-80 geometrical flattening WGS-84 geometrical flattening	$298.257\ 222\ 101^{-1}$ (*published value*) $298.257\ 223\ 563^{-1}$ (*published value*)
b	GRS-80 ellipsoid semi-minor axis WGS-84 ellipsoid semi-minor axis	6,356,752.314 140 m = 20,855,486.594 949 ift = 20,855,444.883 876 sft 6,356,752.314 245 m = 20,855,486.595 293 ift = 20,855,444.884 319 sft
e^2	GRS-80 first eccentricity squared WGS-84 first eccentricity squared	0.006 694 380 022 901 0.006 694 379 990 141
ift	International Foot	1 ift ≡ 0.3048 m (2 ppm shorter than sft)
sft	US Survey Foot	1 sft ≡ 1200/3937 m (2 ppm longer than ift)
ppm	Parts per million	Value multiplied by one million (analogous to "percent" which is "parts per hundred")
rad	Radian (angular measure)	1 rad = 180° / π (i.e., 1 rad ≈ 57.295 779 513°)
π	Pi (irrational number)	π = 3.141 592 653 589 793 238 462 643 383…
γ_0	Normal gravity on the GRS 80 ellipsoid at 45° latitude	9.806199 m/s^2 32.172569 ift/s^2 = 32.172505 sft/s^2

Section 1

GPS, GEODESY, AND THE PERILS OF MODERN POSITIONING

Exercise 1.1: Computation of coordinates from total station data

Total stations determine three-dimensional coordinates by measuring three quantities: 1) *slope distance*, 2) *horizontal angle*, and 3) *zenith angle*.

Grid coordinates (northing and easting) and elevation can be computed from a total station using the following formulas (designated as Equation 1.1):

Equation 1.1 Computation of grid coordinates from total station data

$$\boxed{\begin{aligned} N &= N_0 + D_S \cos\alpha \ \sin\nu \\ E &= E_0 + D_S \sin\alpha \ \sin\nu \\ H &= H_0 + D_S \cos\nu + i - r \end{aligned}}$$

where N, E, and H are the northing, easting, and height (elevation) coordinates to be determined

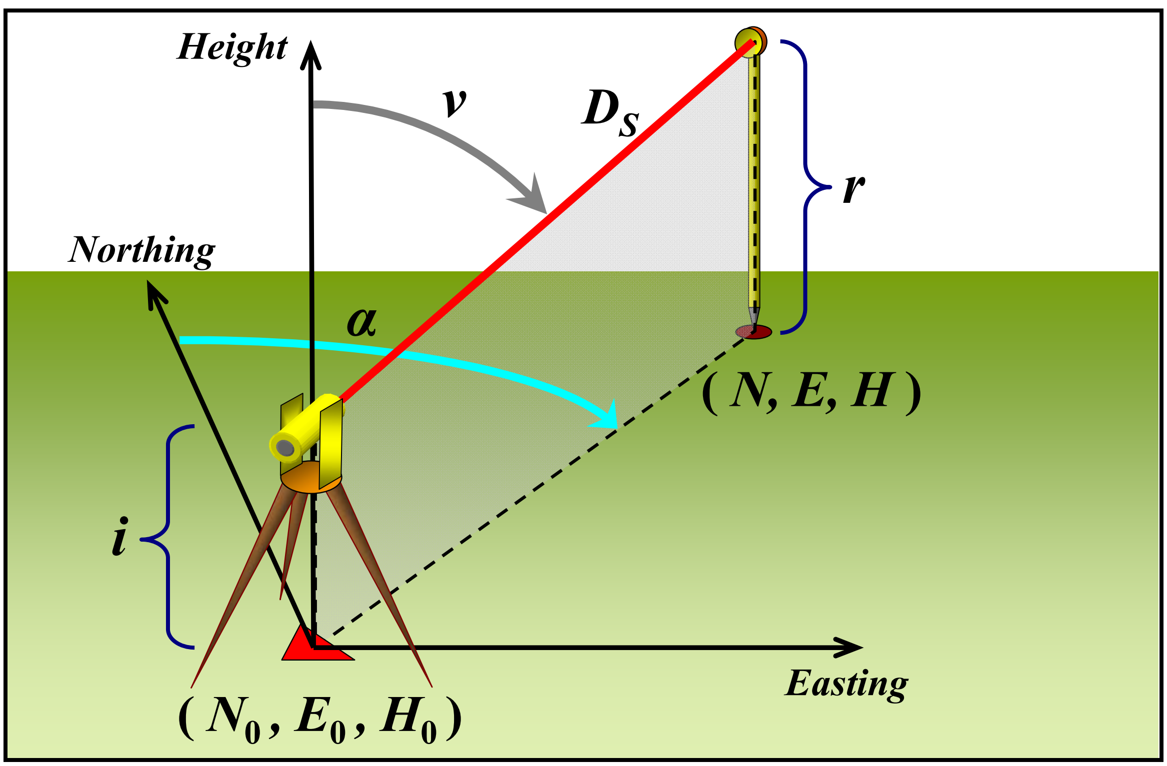
N_0, E_0, and H_0 are the northing, easting, and height of the instrument setup point

D_S is the observed slope distance

α is the observed horizontal angle (azimuth)

ν is the observed zenith angle

i and r are the instrument and the prism rod heights, respectively.



Example computation

Given: A total station set up with i = 5.32 ft over starting point with N_0 = 5000.00 ft, E_0 = 5000.00 ft, and H_0 = 100.00 ft. The horizontal circle is set so that it reads azimuth directly, and the following observations are made to a point with prism rod of height r = 6.56 ft:

$$D_S = 336.84 \text{ ft} \qquad \alpha = 152°17'23'' \qquad v = 83°48'50''$$

Find: The coordinates and elevation of the observed point.

Computations:

$N = \quad N_0 \quad + \quad D_S \quad \times \quad \cos\alpha \quad \times \quad \sin v$

$N =$ ____________ + ______________ × cos(________________) × sin(________________)

$N =$ ____________ + ______________ × ____________________ × ____________________

***N* =** ________________

$E = \quad E_0 \quad + \quad D_S \quad \times \quad \sin\alpha \quad \times \quad \sin v$

$E =$ ____________ + ______________ × sin(________________) × sin(________________)

$E =$ ____________ + ______________ × ____________________ × ____________________

***E* =** ________________

$H = \quad H_0 \quad + \quad D_S \quad \times \quad \cos v \quad + \quad i \quad - \quad r$

$H =$ ____________ + ______________ × cos(________________) + __________ − __________

$H =$ ____________ + ______________ × ____________________ + __________ − __________

***H* =** ________________

Solution:

$N = 5000.00 + 336.84 \times \cos(152°17'23'') \times \sin(83°48'50'')$

$N = 5000.00 + 336.84 \times (-0.88531023) \times 0.99417711$

***N* = 4703.53 ft**

$E = 5000.00 + 336.84 \times \sin(152°17'23'') \times \sin(83°48'50'')$

$E = 5000.00 + 336.84 \times 0.4650009 \times 0.99417711$

***E* = 5155.72 ft**

$H = 100.00 + 336.84 \times \cos(83°48'50'') + 5.32 - 6.56$

$H = 100.00 + 336.84 \times 0.10775836 + 5.32 - 6.56$

***H* = 135.06 ft**

GPS: A geodetic tool

A comparison between total stations and GPS

Both GPS and total stations determine three-dimensional coordinates, but they differ in virtually every other respect, to wit:

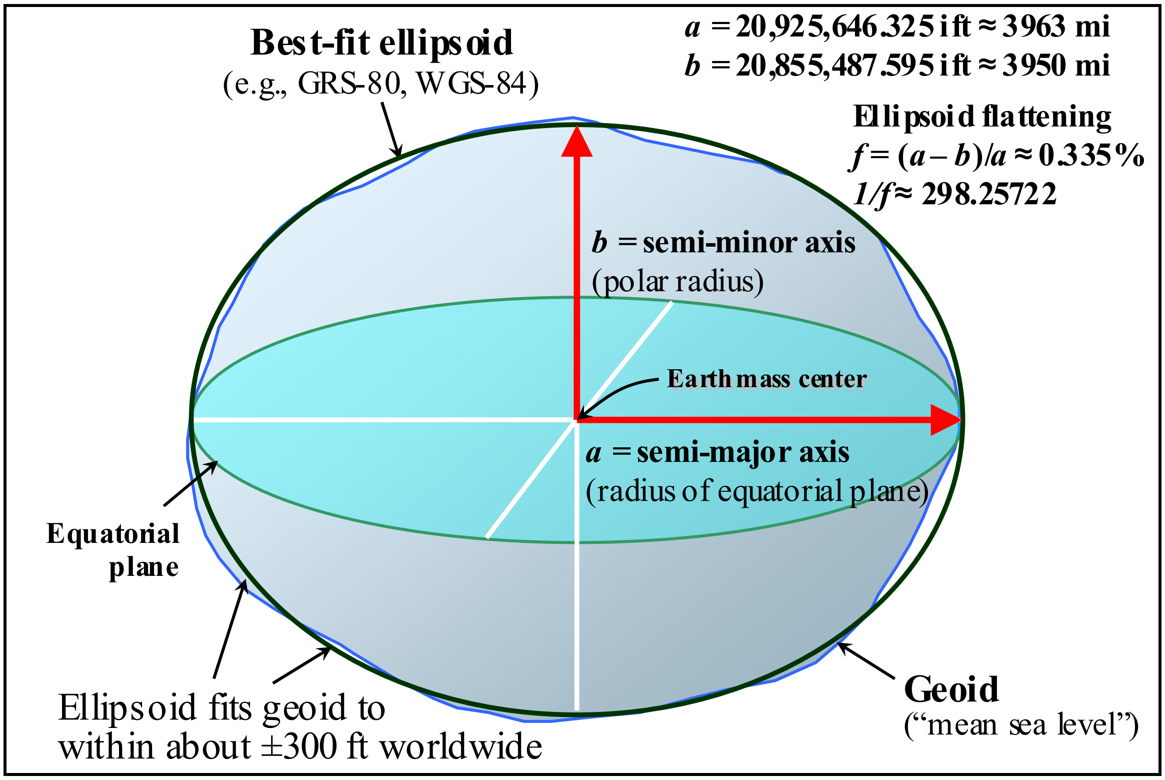
- **Observations**
  - Total stations are used to directly observe slope distance, horizontal angle, and zenith angle
    - Total station EDM sends and receives the signal that it uses for computing distance
  - GPS observes the pseudorange, carrier phase (fractional wavelength), and Doppler shift of the signals transmitted from the satellites
    - GPS only receives signals from the satellites (a one-way ranging system)
- **Measurements**
  - The vector components from a total station to the prism are directly measured
    - Total station measures both distance and angles
  - The vector components between GPS antennas are computed, NOT observed
    - This has implications for error propagation and control network design
    - GPS does NOT measure angles
- **Computations**
  - Coordinates can be determined from total station observations using simple plane trigonometry
  - Geodetic methods MUST be used to compute coordinates from GPS vectors
- **Reference frame**
  - Total stations are referenced to the gravity vector (plumbline) passing through the vertical axis of the instrument
  - GPS is referenced to a world-wide coordinate system (in common with the satellites) with its origin located at the Earth's center of the mass

Geodesy: The science of positioning

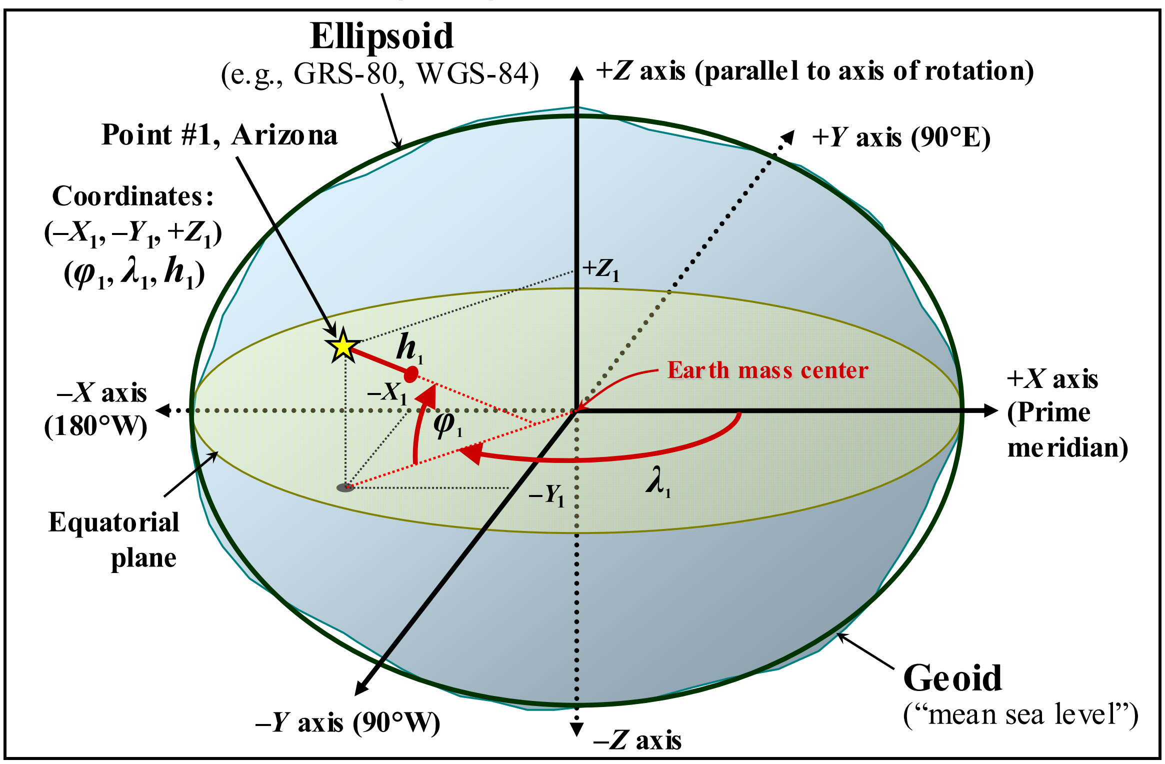
Geodesy is a quantitative scientific field dealing with the size and shape of the Earth (or other planetary bodies), precise determination of coordinates and relationship between coordinates on the Earth, and includes study of the Earth's gravity field. It is the science behind surveying, mapping, and navigation, and it is essential for using GPS.

The bottom line: GPS is a geodetic tool that *requires* geodesy to perform computations and it is explicitly referenced to the *entire* Earth.

The geodetic ellipsoid of revolution



Earth-Centered, Earth-Fixed (ECEF) Cartesian coordinates



Exercise 1.2: Geodetic ellipsoid parameters and computations

The geodetic ellipsoid of revolution is completely defined by two numbers. By convention, these are usually a, the semi-major axis, and $1/f$, the inverse geometric flattening. These can be used to compute other commonly used ellipsoid parameters, such as the following two:

Equation 1.2 Ellipsoid semi-minor axis

$$\boxed{b = a\left(1 - f\right)}$$

Equation 1.3 Ellipsoid first eccentricity squared

$$\boxed{e^2 = 2f - f^2}$$

Example computations

Given: The following parameters for the GRS-80, WGS-84, and Clarke 1866 ellipsoids:

Ellipsoid	GRS-80	WGS-84	Clarke 1866
Semi-major axis, *a*	6,378,137 m (exact)	6,378,137 m (exact)	20,925,832.164 sft
Inverse flattening, 1/*f*	298.257 222 101	298.257 223 563	294.978 698 214

Find: The semi-minor axis (in International Feet) of these ellipsoids.

Computations:

Semi-minor axis = $a \quad \times \quad \left(1 - f\right) \quad \times$ unit conversion

GRS-80: $b = \underline{\hspace{4cm}} \times \left[1 - \left(\dfrac{1}{\underline{\hspace{4cm}}}\right)\right] \times \left(\dfrac{1 \text{ ift}}{0.3048 \text{ m}}\right)$

$b = \underline{\hspace{5cm}}$ **ift**

WGS-84: $b = \underline{\hspace{4cm}} \times \left[1 - \left(\dfrac{1}{\underline{\hspace{4cm}}}\right)\right] \times \left(\dfrac{1 \text{ ift}}{0.3048 \text{ m}}\right)$

$b = \underline{\hspace{5cm}}$ **ift**

Clarke 1866: $b = \underline{\hspace{4cm}} \times \left[1 - \left(\dfrac{1}{\underline{\hspace{4cm}}}\right)\right] \times \left(\dfrac{1.000\,002 \text{ ift}}{1 \text{ sft}}\right)$

$b = \underline{\hspace{5cm}}$ **ift**

Solution:

GRS-80: $b = 6{,}378{,}137 \text{ m} \times \left[1 - \dfrac{1}{298.257222101}\right] \times \left(\dfrac{1 \text{ ift}}{0.3048 \text{ m}}\right) = \underline{\mathbf{20{,}855{,}486.5949 \text{ ift}}}$

WGS-84: $b = 6{,}378{,}137 \text{ m} \times \left[1 - \dfrac{1}{298.257223563}\right] \times \left(\dfrac{1 \text{ ift}}{0.3048 \text{ m}}\right) = \underline{\mathbf{20{,}855{,}486.5953 \text{ ift}}}$

Clarke 1866: $b = 20{,}925{,}832.164 \text{ sft} \times \left[1 - \dfrac{1}{294.978698214}\right] \times \dfrac{1.000002 \text{ ift}}{1 \text{ sft}} = \underline{\mathbf{20{,}854{,}933.727 \text{ ift}}}$

Exercise 1.3: Computation of Earth radius

The Radii of curvature at a point in the *meridian* (north-south) and *prime vertical* (east-west) are frequently used in geodesy:

Equation 1.4 Meridian radius (north-south)

$$\boxed{R_M = \frac{a\left(1-e^2\right)}{\left(1-e^2\sin^2\varphi\right)^{3/2}}}$$

Equation 1.5 Prime vertical radius (east-west)

$$\boxed{R_N = \frac{a}{\sqrt{1-e^2\sin^2\varphi}}}$$

where φ is the geodetic latitude at the point where the radius is computed.

a is the ellipsoid semi-major axis (= 20,925,646.325 459 ift for the GRS-80 ellipsoid)

e^2 is the ellipsoid first eccentricity squared (= 0.006 694 380 022 901 for GRS-80)

R_M and R_N are used to compute other commonly used Earth radii, such as the following two:

Equation 1.6 Radius of curvature in a specific azimuth, α

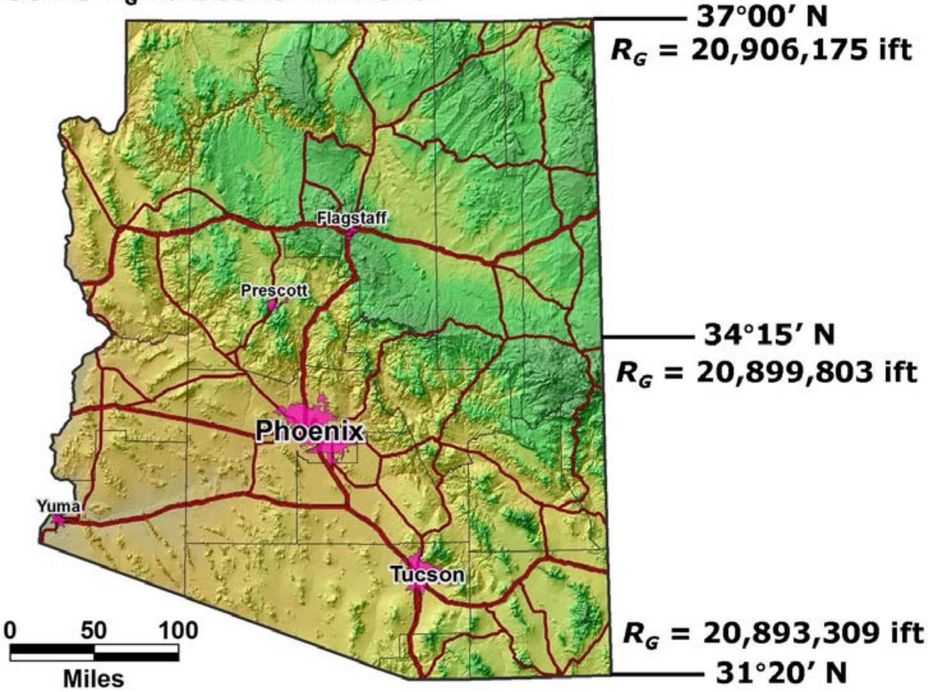
$$\boxed{R_\alpha = \frac{R_M\ R_N}{R_M\sin^2\alpha + R_N\cos^2\alpha}}$$

Equation 1.7 Geometric mean radius of curvature

$$\boxed{R_G = \sqrt{R_M\ R_N} = \frac{a\sqrt{1-e^2}}{1-e^2\sin^2\varphi}}$$

R_G is the essentially the "average" radius of curvature at a point on the ellipsoid, and is the one we will use for radius computations in this workshop.

Some R_G values for Arizona:



Rule of thumb:

Geometric mean radius of curvature *increases* by about 33 ft per mile north (between latitudes of 30° and 40°)

Example computation

Given: A point at latitude φ = 34°32'59.29087" N (midway between points CAS-2 and CAS-3).

Find: The radii of curvature in the meridian, prime vertical, at an (approximate) azimuth of α = 72°06'17" (from CAS-2 to CAS-3), and the geometric mean radius (for the GRS-80 ellipsoid).

Computations: First convert latitude and azimuth to decimal degrees:

$\varphi = 34 + 32/60 + 59.29087/3600 = 34.5498030194°$

$\alpha = 72 + 6/60 + 17/3600 = 72.10473°$

Now compute following function of latitude (since it appears in most of the equations):

$1 - e^2 \sin^2\varphi = 1 - 0.006694380023 \times [\sin(34.5498030194°)]^2 = 0.99784690136$

Now compute the various radii:

$$R_M = \frac{\underline{\hspace{4cm}} \times \left(1 - \underline{\hspace{3cm}}\right)}{\left(\underline{\hspace{3cm}}\right)^{3/2}} = \underline{\hspace{4cm}}$$

$$R_N = \frac{\underline{\hspace{4cm}}}{\sqrt{\underline{\hspace{3cm}}}} = \underline{\hspace{4cm}}$$

$$R_\alpha = \frac{\underline{\hspace{3cm}} \times \underline{\hspace{3cm}}}{\underline{\hspace{3cm}} \times [\sin(\underline{\hspace{1.5cm}})]^2 + \underline{\hspace{3cm}} \times [\cos(\underline{\hspace{1.5cm}})]^2}$$

$$= \underline{\hspace{4cm}}$$

$$R_G = \frac{\underline{\hspace{4cm}} \times \sqrt{1 - \underline{\hspace{3cm}}}}{\underline{\hspace{4cm}}} = \underline{\hspace{4cm}}$$

Solution:

$$R_M = \frac{20{,}925{,}646.325 \times (1 - 0.006694380023)}{(0.99784690136)^{3/2}} = \mathbf{\underline{20{,}852{,}873.272\ ift}}$$

$$R_N = \frac{20{,}925{,}646.325}{\sqrt{0.99784690136}} = \mathbf{\underline{20{,}948{,}210.259\ ift}}$$

$$R_\alpha = \frac{20{,}852{,}872.616 \times 20{,}948{,}210.040}{20{,}852{,}872.616 \times [\sin(72.10473°)]^2 + 20{,}948{,}210.040 \times [\cos(72.10473°)]^2}$$

$$= \mathbf{\underline{20{,}939{,}171.046\ ift}}$$

$$R_G = \frac{20{,}925{,}646.325 \times \sqrt{1 - 0.006694380023}}{0.99784690136} = \mathbf{\underline{20{,}900{,}487.406\ ift}}$$

Check: $R_G = \sqrt{R_M\ R_N} = \sqrt{20{,}852{,}873 \times 20{,}948{,}210} = \underline{20{,}900{,}487.406\ \text{ift}}$ ✓

The NGS Datasheet

```
1        National Geodetic Survey,   Retrieval Date = MARCH 29, 2010
 ES0478 ***********************************************************************
 ES0478  DESIGNATION -  R 18
 ES0478  PID         -  ES0478
 ES0478  STATE/COUNTY-  AZ/YAVAPAI
 ES0478  USGS QUAD   -  CORNVILLE (1968)
 ES0478
 ES0478                         *CURRENT SURVEY CONTROL
 ES0478  ___________________________________________________________________
 ES0478* NAD 83(2007)-  34 43 41.84339(N)    111 58 50.37120(W)     ADJUSTED
 ES0478* NAVD 88     -      1026.381  (meters)    3367.39   (feet)  ADJUSTED
 ES0478  ___________________________________________________________________
 ES0478  EPOCH DATE  -        2007.00
 ES0478  X           -  -1,964,472.392 (meters)                     COMP
 ES0478  Y           -  -4,866,969.363 (meters)                     COMP
 ES0478  Z           -   3,613,704.412 (meters)                     COMP
 ES0478  LAPLACE CORR-           0.93  (seconds)                    DEFLEC09
 ES0478  ELLIP HEIGHT-        1000.746 (meters)          (02/10/07) ADJUSTED
 ES0478  GEOID HEIGHT-         -25.63  (meters)                     GEOID09
 ES0478  DYNAMIC HT  -        1025.099 (meters)    3363.18  (feet)  COMP
 ES0478
 ES0478  ------- Accuracy Estimates (at 95% Confidence Level in cm) --------
 ES0478  Type    PID    Designation                      North   East  Ellip
 ES0478  -------------------------------------------------------------------
 ES0478  NETWORK ES0478 R 18                              0.35    0.29   0.98
 ES0478  -------------------------------------------------------------------
 ES0478  MODELED GRAV-     979,351.9    (mgal)                      NAVD 88
 ES0478
 ES0478  VERT ORDER  -  FIRST     CLASS II
 ES0478
 ES0478.The horizontal coordinates were established by GPS observations
 ES0478.and adjusted by the National Geodetic Survey in February 2007.
 ES0478
 ES0478.The datum tag of NAD 83(2007) is equivalent to NAD 83(NSRS2007).
 ES0478.See National Readjustment for more information.
 ES0478.The horizontal coordinates are valid at the epoch date displayed above.
 ES0478.The epoch date for horizontal control is a decimal equivalence
 ES0478.of Year/Month/Day.
 ES0478
 ES0478.The orthometric height was determined by differential leveling and
 ES0478.adjusted in June 1991.
 ES0478
 ES0478.The X, Y, and Z were computed from the position and the ellipsoidal ht.
 ES0478
 ES0478.The Laplace correction was computed from DEFLEC09 derived deflections.
 ES0478
 ES0478.The ellipsoidal height was determined by GPS observations
 ES0478.and is referenced to NAD 83.
 ES0478
 ES0478.The geoid height was determined by GEOID09.
 ES0478
 ES0478.The dynamic height is computed by dividing the NAVD 88
 ES0478.geopotential number by the normal gravity value computed on the
 ES0478.Geodetic Reference System of 1980 (GRS 80) ellipsoid at 45
 ES0478.degrees latitude (g = 980.6199 gals.).
 ES0478
 ES0478.The modeled gravity was interpolated from observed gravity values.
 ES0478
 ES0478;                    North        East     Units Scale Factor Converg.
 ES0478;SPC AZ C     -   413,436.088   207,499.629   MT  0.99990042   -0 02 11.2
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 ES0478;UTM  12      - 3,843,349.858   410,216.925   MT  0.99969935   -0 33 31.3
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 ES0478!             -  Elev Factor  x  Scale Factor =   Combined Factor
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 ES0478!UTM  12      -   0.99984294  x   0.99969935  =   0.99954233
```

The NGS Datasheet (*continued*)

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ES0478                          SUPERSEDED SURVEY CONTROL
ES0478
ES0478  NAD 83(1992)-  34 43 41.84276(N)    111 58 50.37059(W) AD(       ) A
ES0478  ELLIP H (09/30/99) 1000.753  (m)                       GP(       ) 3 1
ES0478  NAVD 88 (09/30/99) 1026.38   (m)         3367.4    (f) LEVELING    3
ES0478  NGVD 29 (??/??/92) 1025.631  (m)         3364.92   (f) ADJ UNCH    1 2
ES0478
ES0478.Superseded values are not recommended for survey control.
ES0478.NGS no longer adjusts projects to the NAD 27 or NGVD 29 datums.
ES0478.See file dsdata.txt to determine how the superseded data were derived.
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ES0478_SETTING: 66 = SET IN ROCK OUTCROP
ES0478_SP_SET: LIMESTONE LEDGE
ES0478_STAMPING: R 18-1931
ES0478_MARK LOGO: USGS-E
ES0478_MAGNETIC: N = NO MAGNETIC MATERIAL
ES0478_STABILITY: A = MOST RELIABLE AND EXPECTED TO HOLD
ES0478+STABILITY: POSITION/ELEVATION WELL
ES0478_SATELLITE: THE SITE LOCATION WAS REPORTED AS SUITABLE FOR
ES0478+SATELLITE: SATELLITE OBSERVATIONS - April 25, 2009
ES0478
ES0478  HISTORY     - Date     Condition        Report By
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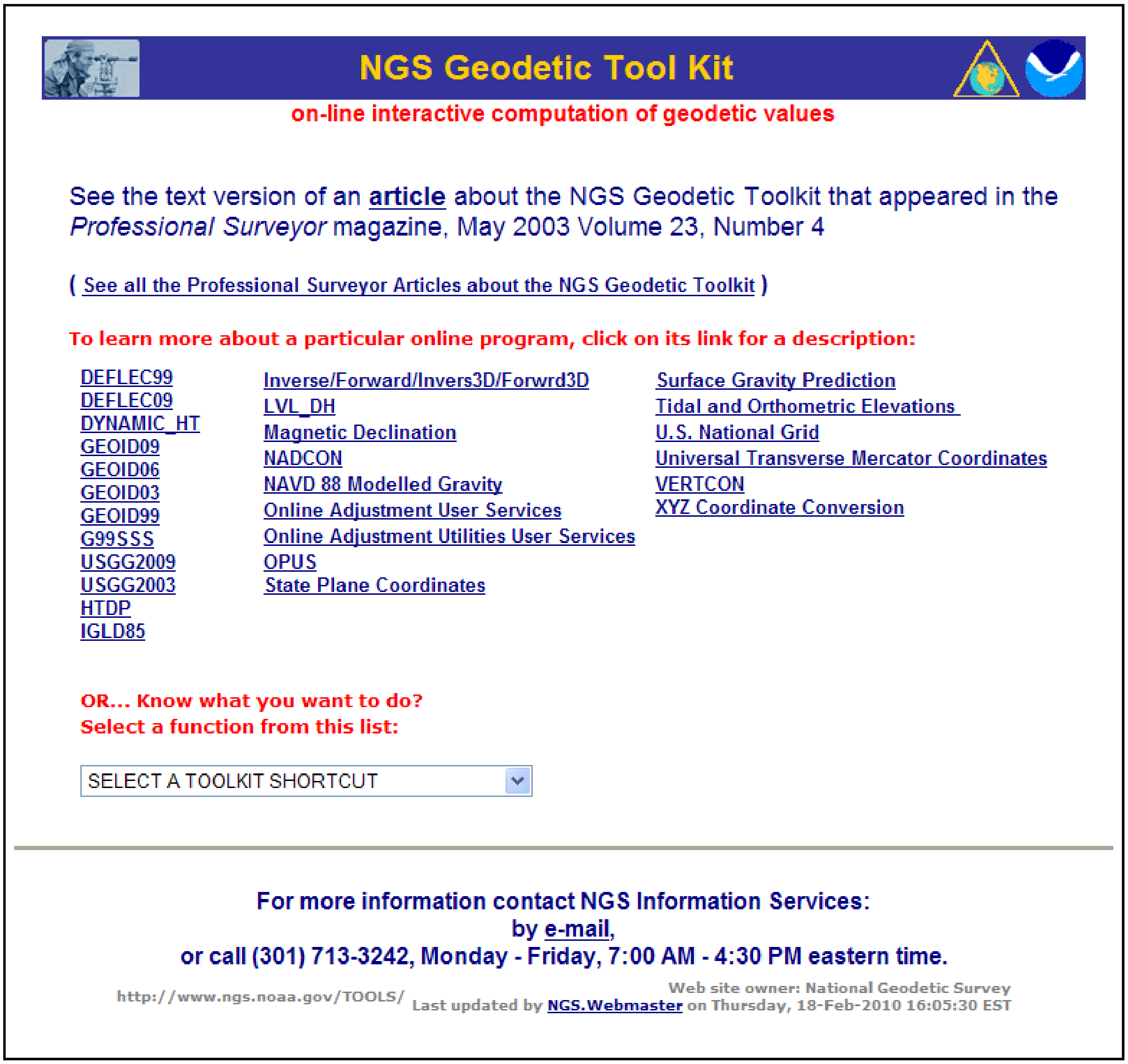
ES0478  HISTORY     - 1933     GOOD             NGS
ES0478  HISTORY     - 19990121 GOOD             AZ-025
ES0478  HISTORY     - 20031215 GOOD             SHEPH
ES0478  HISTORY     - 20090425 GOOD             GEOCAC
ES0478
ES0478                          STATION DESCRIPTION
ES0478
ES0478'DESCRIBED BY NATIONAL GEODETIC SURVEY 1933
ES0478'4.4 MI SE FROM COTTONWOOD.
ES0478'AT SIDE OF HIGHWAY, 0.8 MILES NORTH OF VERDE RIVER HIGHWAY BRIDGE, TOP
ES0478'OF RIDGE, 100 FEET NORTHWEST OF HIGHWAY CENTER-LINE, AT SIDE OF ROCK
ES0478'CAIRN, ON LEDGE OF LIMESTONE PAINTED BLACK U.S.B.M. 3363.9.
ES0478
ES0478                          STATION RECOVERY (1999)
ES0478
ES0478'RECOVERY NOTE BY YAVAPAI COUNTY ARIZONA 1999 (WRA)
ES0478'THE STATION IS LOCATED ABOUT 4.4 MI (7.1 KM) SOUTHEAST OF COTTONWOOD,
ES0478'1 MI (1.6 KM) NORTHEAST OF BRIDGEPORT, 0.8 MI (1.3 KM) NORTHEAST OF A
ES0478'HIGHWAY BRIDGE OVER THE VERDE RIVER, 0.35 MI (0.56 KM) SOUTHWEST OF
ES0478'CORNVILLE ROAD, 0.25 MI (0.40 KM) NORTHEAST OF ROCKING CHAIR ROAD, AT
ES0478'US HIGHWAY 89 ALTERNATE MILEPOST 356.7.  OWNERSHIP--COCONINO NATIONAL
ES0478'FOREST.  TO REACH THE STATION FROM THE JUNCTION OF U.S. HIGHWAY 89
ES0478'ALTERNATE AND STATE HIGHWAY 279 IN COTTONWOOD, GO NORTHEAST FOR 2.2 KM
ES0478'(1.35 MI) ON HIGHWAY 89 ALTERNATE TO THE STATION ON RIGHT, AT THE TOP
ES0478'OF A SMALL KNOLL.  THE STATION IS A DISK SET IN A LIMESTONE OUTCROP.
ES0478'LOCATED 41.5 M (136.2 FT) SOUTHEAST FROM THE CENTERLINE OF HIGHWAY,
ES0478'32.0 M (105.0 FT) NORTHWEST FROM THE CENTER OF AN ABANDONED ROAD, 21.7
ES0478'M (71.2 FT) SOUTHEAST FROM A FENCE, 0.9 M (3.0 FT) SOUTH-SOUTHEAST
ES0478'FROM A ROCK CAIRN AND 0.3 M (1.0 FT) WEST FROM A WITNESS POST.
ES0478
ES0478                          STATION RECOVERY (2003)
ES0478
ES0478'RECOVERY NOTE BY SHEPHARD-WESNITZER INC 2003 (MLD)
ES0478'RECOVERED AS DESCRIBED
ES0478
ES0478                          STATION RECOVERY (2009)
ES0478
ES0478'RECOVERY NOTE BY GEOCACHING 2009 (ACM)
ES0478'RECOVERED IN GOOD CONDITION.
```

The NGS Geodetic Toolkit



NGS Geodetic Tool Kit

on-line interactive computation of geodetic values

See the text version of an article about the NGS Geodetic Toolkit that appeared in the *Professional Surveyor* magazine, May 2003 Volume 23, Number 4

(See all the Professional Surveyor Articles about the NGS Geodetic Toolkit)

To learn more about a particular online program, click on its link for a description:

DEFLEC99
DEFLEC09
DYNAMIC_HT
GEOID09
GEOID06
GEOID03
GEOID99
G99SSS
USGG2009
USGG2003
HTDP
IGLD85

Inverse/Forward/Invers3D/Forwrd3D
LVL_DH
Magnetic Declination
NADCON
NAVD 88 Modelled Gravity
Online Adjustment User Services
Online Adjustment Utilities User Services
OPUS
State Plane Coordinates

Surface Gravity Prediction
Tidal and Orthometric Elevations
U.S. National Grid
Universal Transverse Mercator Coordinates
VERTCON
XYZ Coordinate Conversion

OR... Know what you want to do?
Select a function from this list:

SELECT A TOOLKIT SHORTCUT

For more information contact NGS Information Services:
by e-mail,
or call (301) 713-3242, Monday - Friday, 7:00 AM - 4:30 PM eastern time.

http://www.ngs.noaa.gov/TOOLS/ Web site owner: National Geodetic Survey
Last updated by NGS.Webmaster on Thursday, 18-Feb-2010 16:05:30 EST

Section 2

GEODETIC DATUM DEFINITIONS AND REFERENCE COORDINATES

How are the data connected to the Earth?

Examples of georeferencing errors for Arizona

Table 2.1 Examples of various positioning error sources and their magnitudes for Arizona due to geodetic datum definition and reference coordinate problems (abbreviations and technical terms are defined in the Glossary).

Positioning error examples for Arizona	Error magnitudes
Using NAD 27 when NAD 83 required	Varies from ~210 to 230 feet (horizontal)
Using "WGS 84" when NAD 83 required (e.g., by using WAAS corrections or CORS ITRF coordinates)	~4 feet (horizontal) ~3 feet (vertical)
Using published three-parameter datum transformation between NAD 27 and "WGS 84" for NAD 83 projects	~2 to 16 feet (horizontal)
Using NADCON to transform coordinates between NAD 27 and NAD 83	~1 foot (horizontal)
Using NADCON to transform coordinates between NAD 83(1986) "original" and NAD 83(1992) "HARN"	~0.5 foot (horizontal)
Using NAD 83(1986) "original" when NAD 83(1992) "HARN" required	Up to 3.8 feet (horizontal)
Using NAD 83(1992) "HARN" when CORS or 1999 Arizona FBN unpublished coordinates required	Up to 0.2 foot (horizontal and vertical)
Using NAD 83(1992) "HARN" when NAD 83(NSRS2007) "National Readjustment" coordinates required	Up to 0.5 ft (horizontal) Up to 0.7 ft (vertical)
Using published NGS 14-parameter transformation between "WGS 84" and NAD 83 (CORS) but ignoring velocities and reference (zero) time of 1997	~0.6 ft (horizontal) for coordinates in year 2007
Using reference coordinates found in the header records of CORS raw GPS data files	Varies from zero to over 100 feet (horizontal and vertical)
Autonomous (uncorrected) GPS single-point positioning precision (at 95% confidence)	~10 to 20 ft (horizontal) ~20 to 50 ft (vertical)

The NGS Datasheet as a geodetic reference coordinate source

Recommend using Datasheets with GPS-derived coordinates, because they give ellipsoid height (as well as ECEF coordinates).

```
FQ0454 ***********************************************************************
FQ0454  FBN          -  This is a Federal Base Network Control Station.
FQ0454  DESIGNATION -  FLAGSTAFF NCMN
FQ0454  PID          -  FQ0454
FQ0454  STATE/COUNTY-  AZ/COCONINO
FQ0454  USGS QUAD    -  FLAGSTAFF WEST (1983)
FQ0454
FQ0454                          *CURRENT SURVEY CONTROL
FQ0454  ___________________________________________________________________
FQ0454* NAD 83(2007)-  35 12 52.88846(N)    111 38 05.04201(W)     ADJUSTED
FQ0454* NAVD 88      -      2168.480  (meters)    7114.42   (feet)  ADJUSTED
FQ0454  ___________________________________________________________________
FQ0454  EPOCH DATE   -        2007.00
FQ0454  X            -  -1,923,992.178 (meters)                     COMP
FQ0454  Y            -  -4,850,855.836 (meters)                     COMP
FQ0454  Z            -   3,658,589.263 (meters)                     COMP
FQ0454  LAPLACE CORR-           -2.94  (seconds)                    DEFLEC09
FQ0454  ELLIP HEIGHT-        2145.372 (meters)          (02/10/07) ADJUSTED
FQ0454  GEOID HEIGHT-          -23.15  (meters)                     GEOID09
FQ0454  DYNAMIC HT   -        2165.393 (meters)    7104.29   (feet) COMP
FQ0454
FQ0454  ------- Accuracy Estimates (at 95% Confidence Level in cm) --------
FQ0454  Type     PID    Designation                      North   East  Ellip
FQ0454  -------------------------------------------------------------------
FQ0454  NETWORK FQ0454 FLAGSTAFF NCMN                      0.16    0.14   0.35
FQ0454  -------------------------------------------------------------------
FQ0454  MODELED GRAV-     979,132.0    (mgal)                       NAVD 88
FQ0454
FQ0454  VERT ORDER   -  FIRST     CLASS II
```

= φ and λ

= X

= Y

= Z

= h

Some things to note about NGS Datasheets:

- Units of "feet" in NGS Datasheets are presently US Survey Feet, e.g., for above Datasheet:

  NAVD 88 H = 2168.480 m = 7114.42 sft = 7114.44 ift

- Many conventional stations do not have accurate elevations, so cannot be used with geoid model to determine accurate ellipsoid heights
- Conventionally (optically) determined control is almost always less accurate than survey-grade GPS, so using such control for surveys is not advised
  - Only GPS stations included in the NSRS2007 readjustment have positional accuracies given as linear "network" values in centimeters (relative "order" system not used)
- Epoch date may not be same as CORS ("Continuously Operating Reference Station")
  - NGS station coordinates were determined in 2007 ("NSRS2007")
    - 2007.00 epoch date is used for tectonically active states (California, Arizona, Nevada, Oregon, Washington, and Alaska)
    - 2002.00 epoch date is used for all other states (consistent with CORS epoch date)

OPUS output as a geodetic reference coordinate source

The *Online Positioning User Service*

This is an excellent alternative to the NGS Datasheets if there are no high-quality GPS-derived NGS control stations locally available.

- More accurate than conventional (optical) control
- Requires logging raw GPS data (observables) at the receiver for at least 2 hours (or as little as 15 minutes using the "Rapid Static" option)
    - This can easily be done at a GPS base while performing a survey

```
FILE: cas1_160a.dat 000083824

                              NGS OPUS SOLUTION REPORT
                              ========================

All computed coordinate accuracies are listed as peak-to-peak values.
For additional information: http://www.ngs.noaa.gov/OPUS/about.html#accuracy

      USER: mld@geodeticanalysis.com                 DATE: March 30, 2010
RINEX FILE: cas1160b.05o                             TIME: 04:33:52 UTC

  SOFTWARE: page5  0909.08 master23.pl 081023      START: 2005/06/09  01:40:00
 EPHEMERIS: igs13264.eph [precise]                  STOP: 2005/06/09  06:14:00
  NAV FILE: brdc1600.05n                        OBS USED: 10816 / 11191   :  97%
  ANT NAME: TRM41249.00     NONE             # FIXED AMB:    46 /    51   :  90%
ARP HEIGHT: 2                               OVERALL RMS: 0.010(m)

 REF FRAME: NAD_83(CORS96)(EPOCH:2002.0000)           ITRF00 (EPOCH:2005.4361)

         X:     -2008522.841(m)   0.029(m)          -2008523.523(m)   0.029(m)
         Y:     -4861719.061(m)   0.019(m)          -4861717.725(m)   0.019(m)
         Z:      3597805.541(m)   0.009(m)           3597805.469(m)   0.009(m)

       LAT:   34 32 59.94649      0.012(m)        34 32 59.96249      0.012(m)
     E LON:  247 33 10.81227      0.020(m)       247 33 10.76755      0.020(m)
     W LON:  112 26 49.18773      0.020(m)       112 26 49.23245      0.020(m)
    EL HGT:         1666.715(m)   0.029(m)              1665.871(m)   0.029(m)
 ORTHO HGT:         1693.096(m)   0.033(m) [NAVD88 (Computed using GEOID09)]

                        UTM COORDINATES    STATE PLANE COORDINATES
                         UTM (Zone 12)         SPC (0202 AZ C)
Northing (Y) [meters]     3824090.869          393783.900
Easting (X)  [meters]      367235.276          164688.216
Convergence  [degrees]    -0.82074793         -0.30076926
Point Scale                0.99981726          0.99992919
Combined Factor            0.99955575          0.99966764

US NATIONAL GRID DESIGNATOR: 12SUD6723524090(NAD 83)

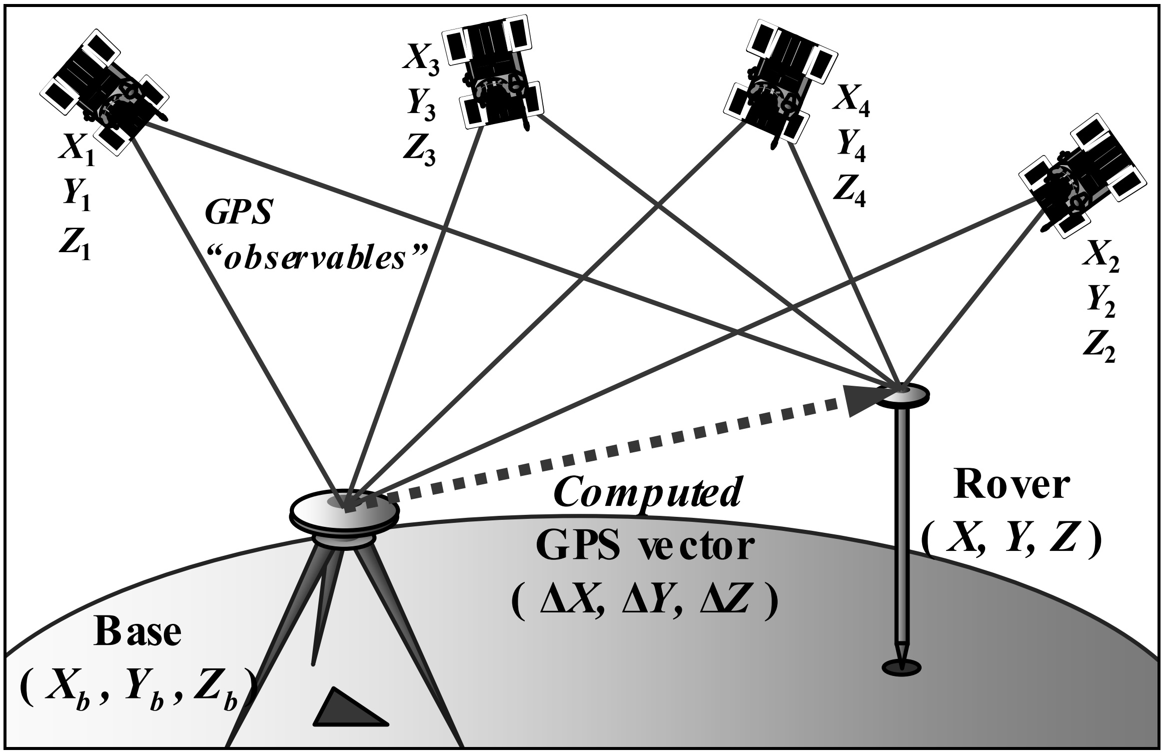

                              BASE STATIONS USED
PID       DESIGNATION                        LATITUDE    LONGITUDE DISTANCE(m)
AI8820 FERN FERNO MESA CORS ARP            N352030.723 W1122717.007   87877.0
AI7445 FST2 FLAGSTAFF 2 CORS ARP           N351317.499 W1114902.781   94167.9
AI7443 FST1 FLAGSTAFF 1 CORS ARP           N351318.370 W1114902.581   94192.2

                 NEAREST NGS PUBLISHED CONTROL POINT
ET0122      D 58                           N343255.    W1122656.        231.3
```

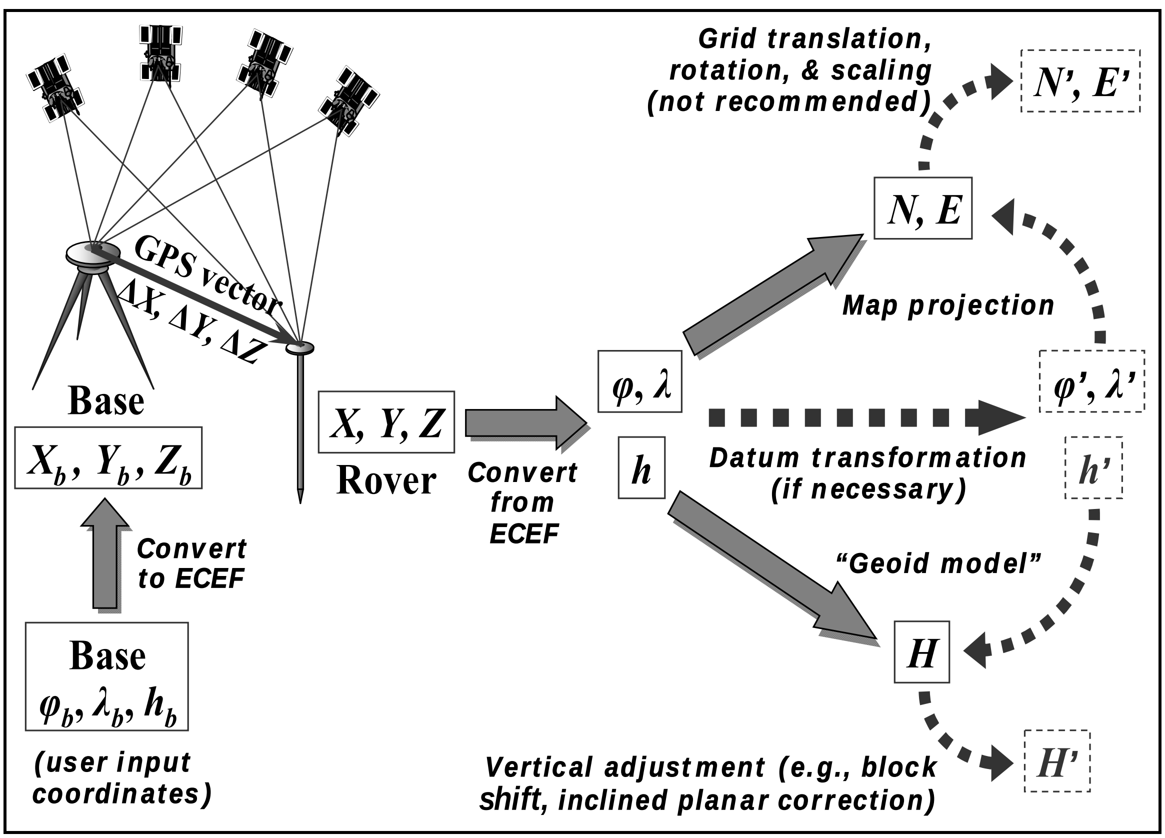
Some things to note about OPUS output:

- Gives both NAD 83 and ITRF 00 coordinates.
  - NAD 83 is for epoch 2002.0 (same as NSRS2007 for most of US, except for California and parts of Arizona, Nevada, Oregon, Washington, and Alaska).
  - ITRF 00 is for day of observation (e.g., date 2005.4361 = June 9, 2005 for this example).
    - This is NOT same as the current version of WGS-84 (G1150), which was computed for a reference time of 2001.0, so the coordinates will differ by the date difference times the ITRF station velocity (about 0.06 ft/year to the SW in AZ, so for this case nearly 0.3 ft).
    - However, ITRF 00 and WGS-84 (G1150) can be considered equivalent to within about 1 – 2 cm (0.03 – 0.07 ft) if both refer to the same reference time.
- Slightly different results will be obtained depending on which GPS orbits were used.
  - Final orbits available after about 2 weeks.
  - “Rapid” orbits available in 17 hours, and are nearly as accurate as final orbits.
- Values to right of coordinates are accuracy estimates in meters, e.g., 0.020 (m).
  - These are based on the maximum difference between the 3 positions computed by OPUS.
  - Can also estimate accuracy (or at least precision) yourself if have multiple OPUS solutions on a single point.
- Detailed (“extended”) output also available
  - Gives additional information such as CORS details, coordinate transformations, velocities, actual vector components, GPS solution statistics, and internal precision estimates.
- Two versions of OPUS now available: OPUS-S (“Static”) and OPUS-RS (“Rapid Static”).
  - OPUS-S was formerly simply known as “OPUS” and requires a dataset duration of at least 2 hours.
  - OPUS-RS will process shorter datasets (duration from 15 minutes to 2 hours).
    - Accuracy of OPUS-RS results varies by location and is best in areas with dense CORS coverage.
    - If poor results are achieved with OPUS-RS, use of OPUS-S is recommended (for dataset durations of more than 2 hours).

Relative positioning with "survey-grade" GPS



GPS computation flowchart



Exercise 2.1: Computation of coordinates using GPS vector components

Below are equations for computing geodetic coordinates of a new station using the GPS vector from a base station of known geodetic coordinates.

Equation 2.1 Converting latitude, longitude, and height to ECEF coordinates

$$\boxed{\begin{aligned} X &= (R_N + h)\cos\varphi\ \cos\lambda \\ Y &= (R_N + h)\cos\varphi\ \sin\lambda \\ Z &= \left[R_N\left(1-e^2\right)+h\right]\sin\varphi \end{aligned}}$$

(Leick, 2004, p. 371)

where X, Y, and Z are the ECEF coordinates of a point

φ, λ, and h are the latitude, longitude, and ellipsoid height of the point, respectively

$\boxed{R_N = a\left(1-e^2\sin^2\varphi\right)^{-1/2}}$ is the prime vertical radius of curvature (Leick, 2004, p. 369)

a is the ellipsoid semi-major axis (= 20,925,646.325 459 ift for the GRS-80 ellipsoid)

e^2 is the ellipsoid first eccentricity squared (= 0.006 694 380 022 901 for GRS-80)

Equation 2.2 Computing coordinates from GPS vector components

$$\boxed{X = X_b + \Delta X \qquad Y = Y_b + \Delta Y \qquad Z = Z_b + \Delta Z}$$

where X, Y, and Z are the ECEF coordinates to be determined

X_b, Y_b, and Z_b are the ECEF coordinates of the GPS base

ΔX, ΔY, and ΔZ are the delta ECEF components of the GPS vector

Equation 2.3 Converting ECEF coordinates to latitude, longitude, and height

$$\boxed{\begin{aligned} \varphi &= \tan^{-1}\left[\frac{Z}{\sqrt{X^2+Y^2}}\left(1+\frac{e^2 R_N \sin\varphi_0}{Z}\right)\right] \\ \lambda &= \tan^{-1}\left(\frac{Y}{X}\right) \\ h &= \frac{\sqrt{X^2+Y^2}}{\cos\varphi} - R_N \end{aligned}}$$

(Leick, 2004, pp. 371-372)

where φ_0 is a latitude that can be *initially* approximated as $\varphi_0 = \tan^{-1}\left(\dfrac{Z}{\left(1-e^2\right)\sqrt{X^2+Y^2}}\right)$.

This approximate latitude value is then substituted into the right side of the first line of Equation 2.3, and then the resulting value of φ is substituted as φ_0, and the process repeated until the change in φ is negligible.

Example computation

Given: A GPS base station located at midpoint between points CAS-2 and CAS-3, with NAD 83 coordinates of φ = 34°32'59.29087" N, λ = 112°26'45.18607" W, and h = 5456.421 ift. The following GPS vector components were determined from this base to point CAS-2:

$\Delta X = -219.000$ ift $\Delta Y = 38.340$ ift $\Delta Z = -51.528$ ift

Find: The NAD 83 coordinates of point CAS-2.

Computations:

Step 1. Convert GPS base latitude, longitude, and ellipsoid height to ECEF coordinates.

The prime vertical radius of curvature for this station was computed in Exercise 1.3:

R_N = ________________

Now compute the ECEF values for the GPS base:

$X_b = (R_N + h) \times \cos\varphi \times \cos\lambda$

X_b = (__________ + ________) × cos(________) × cos(________)

= __________

$Y_b = (R_N + h) \times \cos\varphi \times \sin\lambda$

Y_b = (__________ + ________) × cos(________) × sin(________)

= __________

$Z_b = [R_N \times (1 - e^2) + h] \times \sin\varphi$

Z_b = [__________ × (1− ________) + ________] × sin(________)

= __________

Step 2. Compute ECEF coordinates of new GPS station (CAS-2).

$X = X_b + \Delta X$ = __________ + ________ = __________

$Y = Y_b + \Delta Y$ = __________ + ________ = __________

$Z = Z_b + \Delta Z$ = __________ + ________ = __________

Step 3. Convert ECEF coordinates of new station to latitude, longitude, and ellipsoid height.

Instead of using iterative Equation 2.3, perform this computation using the NGS Geodetic Toolkit, which gives the following coordinates for CAS-2:

Latitude, φ = ____° ____' ____.________" **N**

Longitude, λ = ____° ____' ____.________" **W**

Ellipsoid height, h = ________ **ift**

These results can be verified using Equation 2.3

Solution:

Step 1. Convert GPS base latitude, longitude, and ellipsoid height to ECEF coordinates.

The prime vertical radius of curvature for this station was computed in Exercise 1.3:

$R_N = \underline{20{,}948{,}210.259 \text{ ift}}$

Now compute the ECEF values for the GPS base:

$$X_b = (R_N + h) \times \cos\varphi \times \cos\lambda$$

$$X_b = (20{,}948{,}210.259 + 5456.421) \times \cos(34.5498030194°) \times \cos(-112.4458850194°)$$

$$= \mathbf{\underline{-6{,}589{,}343.061 \text{ ift}}}$$

$$Y_b = (R_N + h) \times \cos\varphi \times \sin\lambda$$

$$Y_b = (20{,}948{,}210.259 + 5456.421) \times \cos(34.5498030194°) \times \sin(-112.4458850194°)$$

$$= \mathbf{\underline{-15{,}950{,}675.460 \text{ ift}}}$$

$$Z_b = \left[R_N \times (1 - e^2) + h\right] \times \sin\varphi$$

$$Z_b = \left[20{,}948{,}210.259 \times (1 - 0.006694380023) + 5456.421\right] \times \sin(34.5498030194°)$$

$$= \mathbf{\underline{11{,}803{,}762.654 \text{ ift}}}$$

Step 2. Compute ECEF coordinates of new GPS station (CAS-2).

$$X = X_b + \Delta X = (-6{,}589{,}343.061 \text{ ift}) + (-219.000 \text{ ift}) = \mathbf{\underline{-6{,}589{,}562.061 \text{ ift}}}$$

$$Y = Y_b + \Delta Y = (-15{,}950{,}675.460 \text{ ift}) + (38.340 \text{ ift}) = \mathbf{\underline{-15{,}950{,}637.120 \text{ ift}}}$$

$$Z = Z_b + \Delta Z = (11{,}803{,}762.654 \text{ ift}) + (-51.528 \text{ ift}) = \mathbf{\underline{11{,}803{,}711.126 \text{ ift}}}$$

Step 3. Convert ECEF coordinates of new station to latitude, longitude, and ellipsoid height.

Equation 2.3 was used to compute the following results for station CAS-2 (compare to those computed using the NGS Geodetic Toolkit).

Latitude, φ = 34° 32' 58.60097" N

Longitude, λ = 112° 26' 47.78016" W

Ellipsoid height, h = 5466.883 ift

✓ *These results were computed using Equation 2.3 (required only 2 iterations in Excel for accuracy shown)*

Datums and datum transformations

Datum. Any quantity or set of quantities used as a reference or basis for determining other quantities.

Geodetic datum. A set of (at least 8) constants specifying the coordinate system for geodetic control (latitude, longitude, height).

2 for reference ellipsoid size and shape (usually semi-major axis and flattening)

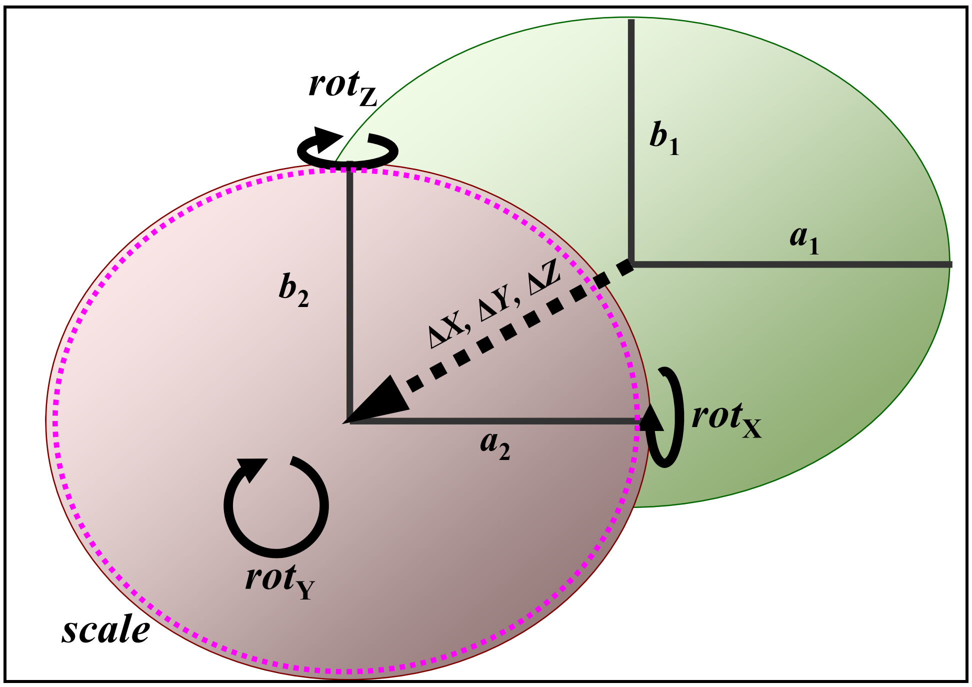
3 to specify location of origin (at or near center of Earth for modern datums)

3 to specify the orientation of coordinate system

Vertical datum. A set of fundamental "elevations" to which other "elevations" are referred.

Datum transformation. Mathematical method for converting one geodetic or vertical datum to another (there are several types, and they vary widely in accuracy).

Geodetic datum transformation



Typical geodetic datum transformations. Note that the dimensions of the reference ellipsoid (*a* and *b* axes) may or may not change in the transformation.

3-parameter: 3-dimensional translation of origin as ΔX, ΔY, ΔZ (*like a GPS vector*)

7-parameter: 3 translations *plus* 3 rotations (one about each of the axes) *plus* a scale

14-parameter: A 7-parameter where each parameter changes with time (each has a *velocity*)

Transformations are also used that model distortion, such as the NGS model NADCON

Vertical datum transformations. Can be a simple vertical shift or a complex operation that models distortion, such as the NGS model VERTCON.

Exercise 2.2: Geodetic azimuths

Forward and reverse grid azimuths differ by exactly 180°. Forward and reverse geodetic azimuths do not differ by 180° because of meridian convergence, as shown in the figure below.

Equation 2.4 Approximate forward geodetic azimuth (from point *A* to point *B*)

$$\boxed{\tilde{\alpha}_{AB} = \tan^{-1}\left(\frac{\lambda_B - \lambda_A}{\varphi_B - \varphi_A}\cos\varphi_B\right)}$$

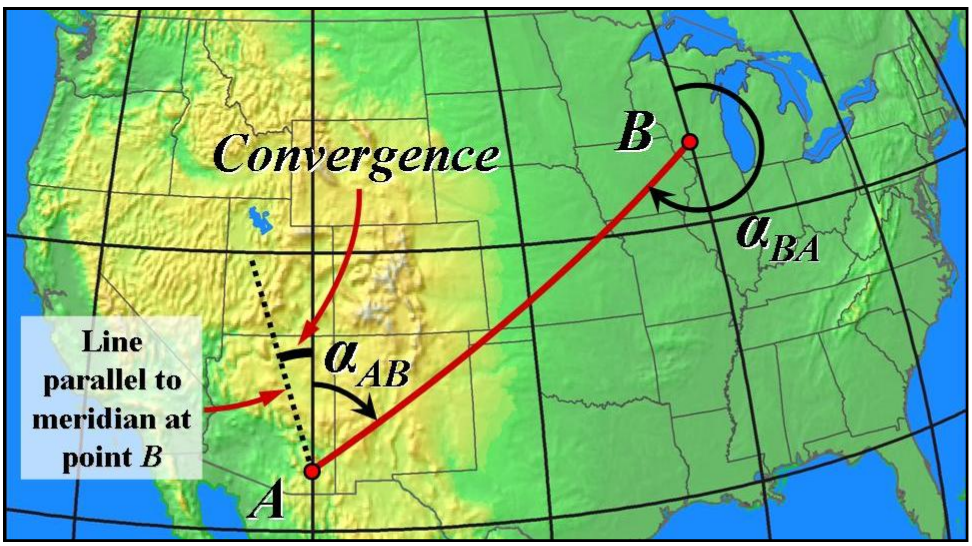
where $\tilde{\alpha}_{AB}$ is the *approximate* forward geodetic azimuth from point *A* to *B*

φ_A, φ_B are longitudes at azimuth end points *A* and *B*, respectively

λ_A, λ_B are longitudes at azimuth end points *A* and *B*, respectively

Equation 2.4 is accurate to within approximately ±0.5% for distances of less than about 100 miles.

Although forward and backward *grid* azimuths differ by exactly 180°, forward and backward geodetic azimuths generally do not due to meridian convergence, as shown in the figure below.



Rule of Thumb:

The average convergence in Arizona is about 35 arc-seconds per mile east-west.

Equation 2.5 Difference between forward and back geodetic azimuths (meridian convergence)

$\boxed{\alpha_{BA} - \alpha_{AB} - 180° \approx (\lambda_B - \lambda_A)\sin\overline{\varphi}}$ (Stem, 1990, p. 51; Ewing and Mitchell, 1970, p. 44)

where α_{AB}, α_{BA} are the forward and back geodetic azimuths from point *A* to *B*, respectively

$\overline{\varphi}$ is the average latitude of the azimuth end points

Although Equation 2.5 is for a sphere, it is accurate to better than 0.2" for distances of less than about 100 miles.

Example computation

Given: A two points (CAS-2 and CAS-3) with the following geodetic coordinates:

CAS-2: φ_A = 34°32'58.60097" N λ_A = 112°26'47.78016" W

CAS-3: φ_B = 34°32'59.98077" N λ_B = 112°26'42.59198" W

Find: The approximate geodetic azimuth from CAS-2 to CAS-3 and compute the difference between the forward and back geodetic azimuths (i.e., the convergence).

Computations:

To simplify the computations, the approximate geodetic azimuth can be computed using the coordinate differences in arc-seconds:

$$\tilde{\alpha}_{AB} \approx \tan^{-1}\left(\frac{\lambda_B - \lambda_A}{\varphi_B - \varphi_A}\cos\varphi_B\right) = \tan^{-1}\left(\frac{(\_\_\_\_\_\_\_\_\_\_)-(\_\_\_\_\_\_\_\_\_\_)}{(\_\_\_\_\_\_\_\_\_\_)-(\_\_\_\_\_\_\_\_\_\_)}\times\cos(\_\_\_\_\_\_\_\_\_\_)\right)$$

$= \tan^{-1}$(____________ × ____________) = __________° = ____° ____' ____" = **N ____° ____' ____" E**

The difference between forward and back azimuths is

$\alpha_{BA} - \alpha_{AB} - 180° \approx (\lambda_B - \lambda_A)\sin\bar{\varphi}$ *(can use midpoint latitude from Exercises 1.3 or 2.1)*

= (____________ + ____________) × sin (____________________)

= ____________ × sin[_______________] = ____________"

Solution:

The approximate geodetic azimuth can be computed as

$$\tilde{\alpha}_{AB} = \tan^{-1}\left(\frac{\lambda_B - \lambda_A}{\varphi_B - \varphi_A}\cos\varphi_B\right) = \tan^{-1}\left(\frac{(-42.59198'')-(-47.78016'')}{(59.98077'')-(58.60097'')}\times\cos(34.5499946583°)\right)$$

$= \tan^{-1}$(3.7600957 × 0.823631645) = **72.10473°** = **72° 06' 17"** = **N 72° 06' 17" E**

The difference between forward and back azimuths is

$\alpha_{BA} - \alpha_{AB} - 180° \approx (\lambda_B - \lambda_A)\sin\bar{\varphi}$ *(can use midpoint latitude from Exercises 1.3 or 2.1)*

= (−42.59198" + 47.78016") × sin(34.5498030194°)

= 5.18818" × 0.5671224 = **+2.9423"**

Check using NGS Inverse tool:

Forward azimuth = 72° 10' 50.3098"

Error in approximate azimuth is −0° 04' 33" = −0.11% *(OK, but not very accurate)* ✓

Back azimuth =252° 10' 53.2521"

Convergence = (252° 10' 53.2521") – (72° 10' 50.3098") –180° = +2.9423" *(the same!)* ✓

Exercise 2.3: An approximate method for computing ellipsoidal distance

This gives a method for computing an approximate ellipsoidal distance between two points with geodetic coordinates (latitude, longitude, and ellipsoid height). For the GRS-80, WGS-84, Clarke 1866, and most other Earth ellipsoids, note the following:

Rules of Thumb

1 arc-second of latitude ≈ 101 ft (*accurate to within about ±0.3 ft in US*)

1 arc-second of longitude ≈ 101 ft × cos(latitude) (*short by about 0.5 ft in US*)

Based on these relationships, we can compute an approximate distance, to wit:

Equation 2.6 Approximate ellipsoidal distance between a pair of geodetic coordinates

$$\boxed{s \approx 101\sqrt{(\Delta\varphi")^2 + (\Delta\lambda"\cos\bar{\varphi})^2}} \text{ feet}$$

This equation is accurate to within about ±1% everywhere on the Earth (and about ±0.5% in AZ)

where $\Delta\varphi"$ is change in latitude between two points in arc-seconds

$\Delta\lambda"$ is change in longitude between two points in arc-seconds

$\bar{\varphi}$ is average latitude of the two points

Example computation

Given: Points CAS-2 and CAS-3 from the previous example (in Exercise 2.2).

Find: The approximate ellipsoidal distance between the points CAS-2 and CAS-3.

Computations:

From the previous example, the average latitude of CAS-2 and CAS-3 is $\bar{\varphi} = 34.5497141306°$

$$s \approx 101\sqrt{(\Delta\varphi")^2 + (\Delta\lambda"\cos\bar{\varphi})^2}$$

$$= 101 \times \sqrt{(\_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_)^2 + [(\_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_) \times \cos(34.5498030194°)]^2}$$

$$= 101 \times \sqrt{(\_\_\_\_\_\_\_\_)^2 + (\_\_\_\_\_\_\_\_)^2} = \underline{\hspace{3cm}\textbf{ft}}$$

Solution:

$$= 101 \times \sqrt{(59.98077 - 58.60097)^2 + [(42.59198 - 47.78016) \times \cos(34.5498030194°)]^2}$$

$$= 101 \times \sqrt{(1.37980)^2 + (-4.27316)^2} = \underline{\textbf{454 ft}}$$

Check using NGS Inverse tool:

Actual ellipsoid distance (geodesic) = 138.9428 m = <u>455.849 ift</u>

✓

Approximate geodetic inverse error = −1.8 ft = −0.4%

Exercise 2.4: A more accurate method for approximating ellipsoidal distance

Computation of accurate geodetic distances is difficult, but a good approximation over short distances can be computed using spherical angles based on an appropriate radius of curvature.

Equation 2.7 Central angle between two points on surface of a sphere

$$\boxed{\psi = \cos^{-1}\left(\sin\varphi_A \, \sin\varphi_B + \cos\varphi_A \, \cos\varphi_B \, \cos(\lambda_A - \lambda_B)\right)}$$

where φ_A, φ_B are the latitudes at points A and B, respectively

λ_A, λ_B are longitudes at points A and B, respectively

Equation 2.8 Approximate geodetic inverse based on spherical angle

$$\boxed{s = R_\alpha \psi = \left(\frac{R_M \; R_N}{R_M \sin^2 \tilde{\alpha}_{AB} + R_N \cos^2 \tilde{\alpha}_{AB}}\right)\psi}$$

where all variables are as defined previously and radii of curvature are evaluated at the mean latitude of the two points.

The accuracy of the distances computed by Equation 2.8 vary with azimuth, and are generally shorter than actual by a maximum of 10 ppm for distances less of than about 10 miles (e.g., a one mile inverse is at most 0.05 ft shorter than actual).

A highly accurate method for computing geodetic distance and azimuth was published by Vincenty (1975), and is the one used in the NGS geodetic tool "Inverse".

Example computation

Given: Points CAS-2 and CAS-3 from the previous two examples (in Exercises 2.2 and 2.3).

Find: The approximate ellipsoidal distance between the points CAS-2 and CAS-3.

Computations:

First compute the spherical angle,

$$\psi = \cos^{-1}\left(\sin\varphi_A \, \sin\varphi_B + \cos\varphi_A \, \cos\varphi_B \, \cos(\lambda_A - \lambda_B)\right)$$

ψ = $\cos^{-1}$[sin(__________________________) × sin(__________________________)

+ cos(__________________________) × cos(__________________________)

× cos(__________________________) − (__________________________)]

= $\cos^{-1}$[(______________________ × ______________________)

+ (______________________ × ______________________ × ______________________)]

= $\cos^{-1}$[____________________ + ____________________] = __________° = __________"

Then compute the ellipsoid distance as (with ψ in radians) as

$$s = R_\alpha \psi = \left(\frac{R_M \; R_N}{R_M \sin^2 \tilde{\alpha}_{AB} + R_N \cos^2 \tilde{\alpha}_{AB}} \right) \psi \,.$$

From Exercise 1.3, the radius of curvature is R_α = 20,939,171 ift (rounded to the nearest foot) at a mean latitude of $\bar{\varphi}$ = 34.5498030194° for points CAS-2 and CAS-3. This is at an (approximate) azimuth of $\tilde{\alpha}$ = 72.10473° from CAS-2 to CAS-3 (Exercises 1.3 and 2.2).

The spherical angle must be converted to radians for this computation, as follows:

$$s = \quad R_\alpha \quad \times \quad \psi$$

$$s = \underline{\hspace{8cm}} \times \underline{\hspace{5cm}}° \times \frac{\pi}{180°} = \underline{\hspace{6cm}\textbf{ift}}$$

<u>Solution</u>:

$$\psi = \cos^{-1}\left(\sin\varphi_A \; \sin\varphi_B + \cos\varphi_A \; \cos\varphi_B \; \cos(\lambda_A - \lambda_B)\right)$$

$$\begin{aligned} \psi \; &= \cos^{-1}\big[\sin(34.5496113805556°) \times \sin(34.5499946583333°) \\ &\qquad + \cos(34.5496113805556°) \times \cos(34.\ 5499946583333°) \\ &\qquad \times \cos(-112.446605600000° + 112.445164438889°)\big] \\ &= \cos^{-1}\big[(0.567119620484369 \times 0.567125130147320) \\ &\qquad + (0.823635438808740 \times 0.823631645066765 \times 0.999999999683663)\big] \\ &= \cos^{-1}\big[0.321627788576296 + 0.678372211186735\big] = \underline{0.0012473344°} = \underline{4.490404''} \end{aligned}$$

The spherical angle must be converted to radians for this computation, as follows:

$$s = 20{,}939{,}171 \text{ ift} \times 0.0012473344° \times \frac{3.14159265}{180°} = \underline{\textbf{455.848 ift}}$$

From Exercise 2.3, Vincenty inverse is nearly identical, s = <u>455.849 ift</u> ✓

(error = −0.001 ft = −0.0002%)

The results shown here were computed using Microsoft *Excel*, which has a numerical precision of 15 digits. Note that most hand calculators have difficulty accurately performing these calculations due to lower numerical precision. Example computations using different numerical precisions are given below (these will vary depending on the calculator, sequence of computations, and number of digits entered):

14 digits of numerical precision ➔ s = 455.845 ift (−0.0009% error)

13 digits of numerical precision ➔ s = 455.821 ift (−0.0061% error)

12 digits of numerical precision ➔ s = 455.611 ift (−0.0522% error)

Exercise 2.5: Deflection of the vertical and the Laplace correction

In general, the plumbline (gravity vector) passing through the axis of an instrument is not parallel to a line perpendicular to the reference ellipsoid (the ellipsoid normal), and the angle between these two lines is called the *deflection of the vertical*. The deflection of the vertical is divided into north-south and east-west components, denoted as ξ and η, respectively. These can be obtained from the NGS model DEFLEC09 and USDOV2009 for any location in the US. DEFLEC09 was derived from the GEOID09 "hybrid" geoid model, and is the appropriate one to use for survey observations referenced to NAD 83. USDOV2009 was derived from the purely gravimetric geoid model USGG2009, which is referenced to ITRF 00.

If the deflection of the vertical is not zero, an instrument leveled to the local plumbline will not be "level" with respect to the geodetic datum. When using terrestrial (optical) instruments, this affects determination of coordinates and azimuths using astronomic (or gyroscopic) methods; reductions of terrestrial observations to the ellipsoid; and change in ellipsoid height. In addition, since deflection of the vertical varies with location, it can cause horizontal and vertical errors in terrestrial surveys that are similar to the misclosure that occurs if a traverse is performed with an improperly leveled instrument.

The *Laplace correction* is the difference between astronomic and geodetic azimuth caused by deflection of the vertical. A simplified version of the Laplace correction is given on NGS datasheets, and *adding* this value to (clockwise) astronomic azimuths will give the geodetic azimuth for an approximately horizontal line of sight between stations.

Equation 2.9 The simplified (horizontal) Laplace correction (assumes approximately horizontal line of sight, a clockwise positive azimuth, and a positive east deflection of the vertical):

$$\boxed{L = \alpha - A = -\eta \tan\varphi}$$

where α and A are the geodetic and astronomic azimuths, respectively

η is the deflection of the vertical component in the east-west (prime vertical) direction

φ is the geodetic latitude

Rules of Thumb

Maximum deflection of the vertical in Arizona = 35 arc-seconds (DEFLEC09)

Maximum Laplace correction magnitude in Arizona = 25 arc-seconds (DEFLEC09)

Simplified Laplace correction error is less than approximately 10% for zenith angles within about 5° of horizontal

Example computation

Given: In Elbow Canyon of the Virgin Mountains of northwestern Arizona, GPS was used to locate the southwest corner and the west quarter corner of Section 16, T 39 N, R 15 W, Gila and Salt River Baseline and Meridian. The following NAD 83 coordinates were obtained:

Station	Latitude	Longitude	Ellipsoid height
SW Corner S16	36°46'31.61284"N	113°55'21.70113"W	3530.589 ift
W 1/4 Corner S16	36°46'57.75891"N	113°55'21.69613"W	3275.291 ift

The geodetic azimuth and horizontal ground distance from the southwest corner to the west quarter corner based on these coordinates is 0° 00' 31.73" and 2644.715 ift.

Find: The astronomic quadrant bearing from the southwest corner to the west quarter corner of Section 16.

Computations:
For the southwest corner of Section 16, DEFLEC09 gives ξ = 9.18", η = −26.48", and L = 19.79". Equation 2.9 can be rearranged to compute the astronomic azimuth:

$A = \alpha - L =$ ________________ − __________ = ________________

Astronomic quadrant bearing = __________________ (rounded to nearest arc-second)

Solution:
For the southwest corner of Section 16, DEFLEC09 gives ξ = 9.18", η = −26.48", and L = 19.79". Equation 2.9 can be rearranged to compute the astronomic azimuth:

$A = \alpha - L =$ 0° 00' 31.73" − 19.18" = 0° 00' 12.55"

Astronomic quadrant bearing = **N 00° 00' 13" E** (rounded to nearest arc-second)

Check: $L = -\eta \tan \varphi$ = −(+26.48") × tan(36°46'32"N) = 19.79", as given by DEFLEC09.

How accurate is the simplified (horizontal) Laplace correction?

The complete Laplace correction is given by $L = -\eta \tan\varphi - (\xi \sin\alpha - \eta\cos\alpha)\cot\zeta$, where the first term is the same as Equation 2.9, and the second term is referred to as the *deflection correction*. The quantity ζ is the geodetic zenith angle, which can be estimated using the ellipsoid height difference and distance between the corners. Earth curvature increases the zenith angle, and can be accounted for by subtracting 0.0239×(distance in thousands of feet)^2 from the height difference (this correction is covered in more detail in Exercise 4.2):

$$\zeta = 90° - \tan^{-1}\{[(3275.291 - 3530.589) - 0.0239\times 2.644715^2] / 2644.715\} = 95.517°.$$

Thus the deflection correction is:

$$[9.18" \times \sin(0°00'31.73") - (-26.48") \times \cos(0°00'31.73")] \times \cot(95.517°) = \underline{-2.56"}$$

This gives a complete Laplace correction of L = 19.79" − (−2.56") = 22.35". Although this deflection correction is rather large, note that this is a worse-case scenario, because the deflection of the vertical value in this example is essentially the maximum for Arizona. In most cases, the deflection correction is smaller than can be resolved using optical methods, and the simplified Laplace correction will suffice. This helps tremendously, since the simplified Laplace correction does not depend on the azimuth or zenith angle between stations, and so a unique value can be specified at a point.

Section 3

GRID COORDINATE SYSTEMS AND COMPUTATIONS

How are the data displayed? How are the data used?

Examples of grid coordinate errors for Arizona

Table 3.1 Examples of various positioning error sources and their magnitudes for Arizona due to grid coordinate system and computation problems (abbreviations and technical terms are defined in the Glossary).

Positioning error examples for Arizona	Error magnitudes
Using SPCS 27 projection parameters for SPCS 83 projects	37.9 ***miles*** (horizontal)
Determining State Plane coordinates in US Survey Feet when International Feet are required	Up to 5 feet (horizontal)
Determining UTM coordinates in US Survey Feet when International Feet are required	Up to 27 feet (horizontal)
Using linear coordinates from a geographic "projection" to compute distances	Up to ~1000 feet horizontal per mile
Using SPCS grid distances when "ground" distances are required (example here is for Flagstaff)	~2.3 feet horizontal per mile at elevation of 7000 feet
Using UTM grid distances when "ground" distances are required (example here is for Flagstaff)	~3.6 feet horizontal per mile at elevation of 7000 feet
Using planar computation methods to transform geodetically-derived horizontal coordinates (example here is for converting from UTM to SPCS over a 20 mi × 20 mi area in Phoenix area using planar scaling, rotation, and translation based on two common points)	Varies, but increases rapidly with size of area (3 to 4 feet of horizontal error for this example)

Grid coordinate system information in NGS Datasheets and OPUS output

Both NGS Datasheets and OPUS output use the geodetic coordinates of the point to compute grid (map projection) coordinates in the State Plane and Universal Transverse Mercator coordinate systems. They also provide the convergence angle, grid point scale factor, and combined scale factor for both systems.

Portion of NGS Datasheet for station PR 23

```
AI1939  FBN          -  This is a Federal Base Network Control Station.
AI1939  DESIGNATION -  PR 23
AI1939  PID          -  AI1939
AI1939  STATE/COUNTY-  AZ/YAVAPAI
AI1939  USGS QUAD    -  PRESCOTT VALLEY SOUTH (1973)
AI1939
AI1939                        *CURRENT SURVEY CONTROL
AI1939  ___________________________________________________________________
AI1939* NAD 83(2007)-  34 34 33.49068(N)    112 17 18.12513(W)    ADJUSTED
AI1939* NAVD 88     -      1482.5    (meters)    4864.     (feet)  GPS OBS
AI1939  ___________________________________________________________________
AI1939  EPOCH DATE  -        2007.00
AI1939  X           -  -1,994,369.018 (meters)                      COMP
AI1939  Y           -  -4,865,587.497 (meters)                      COMP
AI1939  Z           -   3,600,060.612 (meters)                      COMP
AI1939  LAPLACE CORR-           2.23  (seconds)                     DEFLEC09
AI1939  ELLIP HEIGHT-        1456.454 (meters)          (02/10/07)  GPS OBS
AI1939  GEOID HEIGHT-          -26.06 (meters)                      GEOID09
.
.                                                         = k         = γ
AI1939;                        North         East     Units Scale Factor Converg.
AI1939;SPC AZ C     -   396,601.168   179,257.269    MT  0.99991433   -0 12 39.4
AI1939;SPC AZ C     - 1,301,184.93    588,114.40    iFT  0.99991433   -0 12 39.4
AI1939;UTM  12      - 3,826,775.422   381,827.449    MT  0.99977212   -0 43 52.4
AI1939
AI1939!             -  Elev Factor  x  Scale Factor =   Combined Factor
AI1939!SPC AZ C     -   0.99977143  x   0.99991433  =   0.99968578   = δ + 1
AI1939!UTM  12      -   0.99977143  x   0.99977212  =   0.99954360
```

Portion of OPUS output for station CAS-1

```
                          NGS OPUS SOLUTION REPORT
                          ========================
REF FRAME: NAD_83(CORS96)(EPOCH:2002.0000)           ITRF00 (EPOCH:2005.4361)

         X:      -2008522.841(m)   0.029(m)        -2008523.523(m)   0.029(m)
         Y:      -4861719.061(m)   0.019(m)        -4861717.725(m)   0.019(m)
         Z:       3597805.541(m)   0.009(m)         3597805.469(m)   0.009(m)

       LAT:   34 32 59.94649      0.012(m)       34 32 59.96249      0.012(m)
     E LON:  247 33 10.81227      0.020(m)      247 33 10.76755      0.020(m)
     W LON:  112 26 49.18773      0.020(m)      112 26 49.23245      0.020(m)
    EL HGT:         1666.715(m)   0.029(m)             1665.871(m)   0.029(m)
 ORTHO HGT:         1693.096(m)   0.033(m) [NAVD88 (Computed using GEOID09)]

                        UTM COORDINATES    STATE PLANE COORDINATES
                         UTM (Zone 12)         SPC (0202 AZ C)
Northing (Y) [meters]     3824090.869          393783.900
Easting (X)  [meters]      367235.276          164688.216
Convergence  [degrees]    -0.82074793         -0.30076926 = γ
Point Scale                0.99981726          0.99992919   = k
Combined Factor            0.99955575          0.99966764     = δ + 1
```

Map projection distortion

Map projection distortion is an *unavoidable* consequence of attempting to represent a curved surface on a flat surface. It can be thought of as a change in the "true" relationship between points located on the surface of the Earth and the *representation* of their relationship on a plane. Distortion cannot be eliminated — it is a ***F***act of ***L***ife. The best we can do is decrease the effect.

There are two general types of map projection distortion:

1. Linear distortion. Difference in distance between a pair of grid (map) coordinates when compared to the true ("ground") distance, denoted here by $\boldsymbol{\delta}$.
    - Can express as a ratio of distortion length to ground length:
        - E.g., feet of distortion per mile; parts per million (= mm per km)
        - *Note:* 1 foot / mile = 189 ppm = 189 mm / km
    - Linear distortion can be positive or negative:
        - NEGATIVE distortion means the grid (map) length is SHORTER than the "true" horizontal (ground) length.
        - POSITIVE distortion means the grid (map) length is LONGER than the "true" horizontal (ground) length.
2. Angular distortion. For conformal projections (e.g., Transverse Mercator, Lambert Conformal Conic, Stereographic, Oblique Mercator, etc.), it equals the *convergence* (*mapping*) *angle*, $\boldsymbol{\gamma}$. The convergence angle is the difference between grid (map) north and true (geodetic) north.
    - Convergence angle is zero on the projection central meridian, positive east of the central meridian, and negative west of the central meridian
    - Magnitude of the convergence angle increases with distance from the central meridian, and its rate of change increases with increasing latitude:

Latitude	Convergence angle 1 mile from CM	Latitude	Convergence angle 1 mile from CM
0°	0° 00' 00"	50°	±0° 01' 02"
10°	±0° 00' 09"	60°	±0° 01' 30"
20°	±0° 00' 19"	70°	±0° 02' 23"
30°	±0° 00' 30"	80°	±0° 04' 54"
40°	±0° 00' 44"	89°	±0° 49' 32"

- Usually convergence is not as much of a concern as linear distortion, and it can only be minimized by staying close to the projection central meridian (or the Equator).

Total linear distortion of grid (map) coordinates is a combination of distortion due to Earth curvature and distortion due to ground height above the ellipsoid. In many areas, distortion due to variation in ground height is greater than that due to curvature. This is illustrated in the diagrams and tables on the following pages.

Table 3.2 Horizontal distortion of grid coordinates due to Earth curvature

Maximum zone width for secant projections (miles)	**Maximum linear horizontal distortion, δ**		
	Parts per million	**Feet per mile**	**Ratio (absolute value)**
16 miles	±1 ppm	±0.005 ft/mile	1 : 1,000,000
50 miles	±10 ppm	±0.05 ft/mile	1 : 100,000
71 miles	**±20 ppm**	**±0.1 ft/mile**	**1 : 50,000**
112 miles	±50 ppm	±0.3 ft/mile	1 : 20,000
158 miles (e.g., SPCS)*	±100 ppm	±0.5 ft/mile	1 : 10,000
317 miles (e.g., UTM)†	±400 ppm	±2.1 ft/mile	1 : 2500

*State Plane Coordinate System; zone width shown is valid between ~0° and 45° latitude
†Universal Transverse Mercator; zone width shown is valid between ~30° and 60° latitude

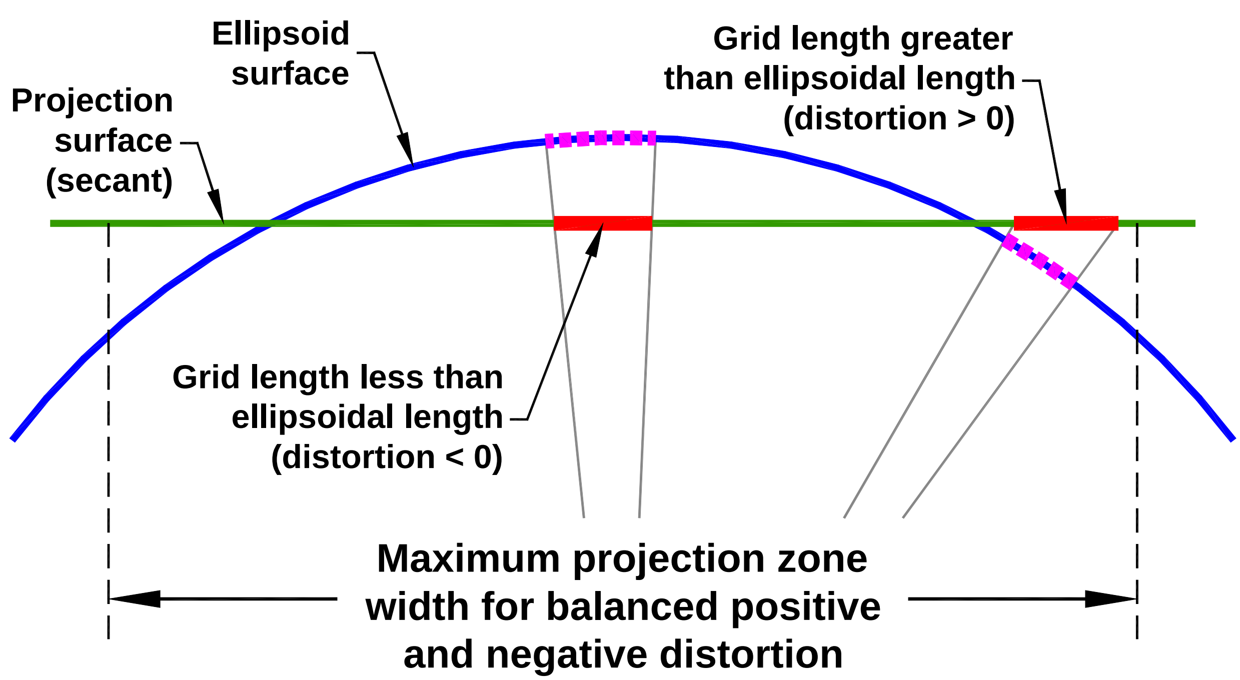


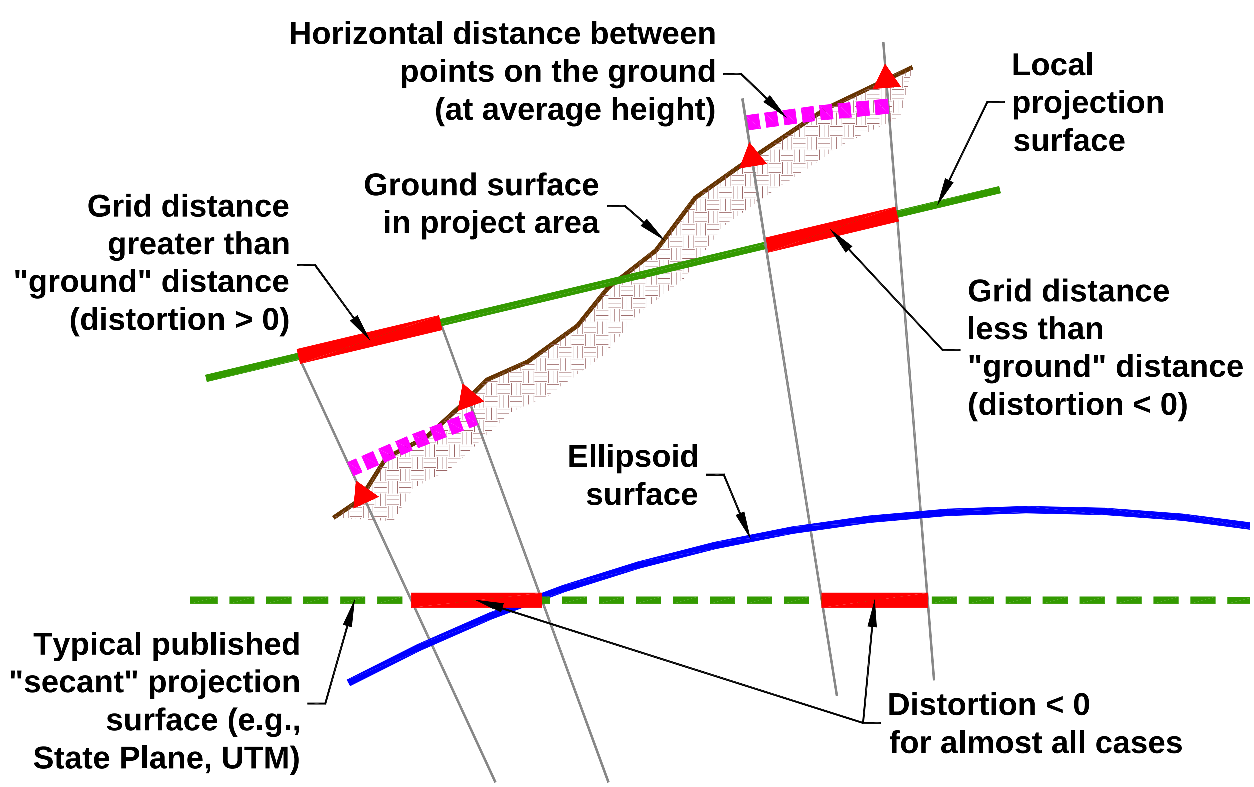
Table 3.3 Horizontal distortion of grid coordinates due to ground height above the ellipsoid

Height below (–) and above (+) projection surface	Maximum linear horizontal distortion, δ		
	Parts per million	Feet per mile	Ratio (absolute value)
–100 feet, +100 feet	±4.8 ppm	±0.03 ft/mile	~1 : 209,000
–400 feet, +400 feet	**±19 ppm**	**±0.1 ft/mile**	**~1 : 52,000**
–1000 feet, +1000 feet	±48 ppm	±0.3 ft/mile	~1 : 21,000
+2000 feet	–96 ppm	–0.5 ft/mile	~1 : 10,500
+4000 feet*	–191 ppm	–1.0 ft/mile	~1 : 5200
+7000 feet**	–335 ppm	–1.8 ft/mile	~1 : 3000
+12,600 feet†	–603 ppm	–3.2 ft/mile	~1 : 1700

*Approximate average topographic height in Arizona
** Approximate topographic height of Flagstaff, Arizona
† Approximate maximum topographic height in Arizona

Rule of Thumb:

A 100-ft change in height causes a 4.8 ppm change in distortion



Exercise 3.1: Distortion computations

Linear distortion is the ratio of grid distance to horizontal ground distance. One way to estimate distortion is to compute the distance between a pair of points based on the grid coordinates determined by the GPS software. This grid distance can then be divided by the ground distance between these points measured using a (properly calibrated) tape or EDM.

Equation 3.1 Approximating distortion at a point using measured grid and ground distances

$$\boxed{\delta \approx \left(\frac{\sqrt{\Delta N^2 + \Delta E^2}}{\text{measured horizontal ground distance}} \right) - 1}$$

Distortion can be computed more accurately (and conveniently) at a single point using the familiar "combined scale factor" approach:

Equation 3.2 Computing distortion at a point using Earth radius

$$\boxed{\delta = k\left(\frac{R_G}{R_G + h} \right) - 1}$$

Example computation

<u>Given</u>: Points CAS-2 and CAS-3 from the previous examples. The ellipsoid heights (h) of these points are listed below, along with the grid coordinates and grid point scale factors (k) derived from the adjusted geodetic coordinates (given in Exercise 2.2). A horizontal ground distance of 455.968 ift was carefully measured between these points.

CAS-2: NAD 83 (2002.0) ellipsoid height, h = 5466.883 ift

Coordinate system	Northing, *N* (ift)	Easting, *E* (ift)	Grid scale factor, *k*
SPCS 83, Arizona Central (0202)	1,291,805.295	540,432.685	0.999 929 147
UTM 83, Zone 12 North	12,546,092.208	1,204,955.902	0.999 817 145
Low Distortion Projection (LDP)	18,061.311	56,042.621	1.000 258 042

CAS-3: NAD 83 (2002.0) ellipsoid height, h = 5445.959 ift

Coordinate system	Northing, *N* (ift)	Easting, *E* (ift)	Grid scale factor, *k*
SPCS 83, Arizona Central (0202)	1,291,942.505	540,867.361	0.999 928 988
UTM 83, Zone 12 North	12,546,225.452	1,205,391.755	0.999 816 711
Low Distortion Projection (LDP)	18,200.930	56,476.686	1.000 258 048

<u>Find</u>: The linear distortion (in parts per million) at the midpoint between points CAS-2 and CAS-3 in SPCS, UTM, and LDP coordinates using both Equations 3.1 and 3.2 (the geometric mean radius of curvature R_G = 20,900,487 ift was determined at the midpoint in Exercise 1.3).

Computations: For midpoint, use the mean grid scale factor and mean ellipsoid height = 5456 ft.

SPCS 83 AZ C

Using Equation 3.1:

$$\delta \approx \left(\frac{\sqrt{(\_\_\_\_\_\_\_\_\_\_\_\_)^2 + (\_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_)^2}}{\_\_\_\_\_\_\_\_} \right) - 1 = \left(\frac{\_\_\_\_}{\_\_\_\_} \right) - 1$$

$= \_\_\_\_\_\_\_\_ - 1$ ➔ *in parts per million* ➔ $\_\_\_\_\_\_\_\_ \times 1{,}000{,}000 = \_\_\_\_\_\_$

Using Equation 3.2:

$$\delta = \frac{\_\_\_\_\_ + \_\_\_\_\_}{2} \left(\frac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_ + \_\_\_\_\_} \right) - 1 = \_\_\_\_\_\_ - 1 \rightarrow \_\_\_\_\_\_$$

UTM 83 12N

Using Equation 3.1:

$$\delta \approx \left(\frac{\sqrt{(\_\_\_\_\_\_\_\_\_\_\_\_)^2 + (\_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_)^2}}{\_\_\_\_\_\_\_\_} \right) - 1 = \left(\frac{\_\_\_\_}{\_\_\_\_} \right) - 1$$

$= \_\_\_\_\_\_\_\_ - 1$ ➔ *in parts per million* ➔ $\_\_\_\_\_\_\_\_ \times 1{,}000{,}000 = \_\_\_\_\_\_$

Using Equation 3.2:

$$\delta = \frac{\_\_\_\_\_ + \_\_\_\_\_}{2} \left(\frac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_ + \_\_\_\_\_} \right) - 1 = \_\_\_\_\_\_ - 1 \rightarrow \_\_\_\_\_\_$$

LDP

Using Equation 3.1:

$$\delta \approx \left(\frac{\sqrt{(\_\_\_\_\_\_\_\_\_\_\_\_)^2 + (\_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_)^2}}{\_\_\_\_\_\_\_\_} \right) - 1 = \left(\frac{\_\_\_\_}{\_\_\_\_} \right) - 1$$

$= \_\_\_\_\_\_\_\_ - 1$ ➔ *in parts per million* ➔ $\_\_\_\_\_\_\_\_ \times 1{,}000{,}000 = \_\_\_\_\_\_$

Using Equation 3.2:

$$\delta = \frac{\_\_\_\_\_ + \_\_\_\_\_}{2} \left(\frac{\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_ + \_\_\_\_\_} \right) - 1 = \_\_\_\_\_\_ - 1 \rightarrow \_\_\_\_\_\_$$

Solution: For midpoint, use the mean grid scale factor and mean ellipsoid height = 5456 ft.

SPCS 83 AZ C

Using Equation 3.1:

$$\delta \approx \left(\frac{\sqrt{(1{,}291{,}942.505 - 1{,}291{,}805.295)^2 + (540{,}867.361 - 540{,}432.685)^2}}{455.968} \right) - 1 = \left(\frac{455.817}{455.968} \right) - 1$$

= 0.9996688 – 1 ➔ *in parts per million* ➔ –0.0003312 × 1,000,000 = **–331.2 ppm**

Using Equation 3.2:

$$\delta = \frac{0.9999291 + 0.9999290}{2} \left(\frac{20{,}900{,}487}{20{,}900{,}487 + 5456} \right) - 1 = 0.9996681 - 1$$ ➔ **–331.9 ppm**

(= –1.75 ft/mile)

UTM 83 12N

Using Equation 3.1:

$$\delta \approx \left(\frac{\sqrt{(12{,}546{,}225.452 - 12{,}546{,}092.208)^2 + (1{,}205{,}391.755 - 1{,}204{,}955.902)^2}}{455.968} \right) - 1 = \left(\frac{455.766}{455.968} \right) - 1$$

= 0.9995570 – 1 ➔ *in parts per million* ➔ –0.0004430 × 1,000,000 = **–443.0 ppm**

Using Equation 3.2:

$$\delta = \frac{0.9998171 + 0.9998167}{2} \left(\frac{20{,}900{,}487}{20{,}900{,}487 + 5456} \right) - 1 = 0.9995560 - 1$$ ➔ **–444.0 ppm**

(= –2.34 ft/mile)

LDP

Using Equation 3.1:

$$\delta \approx \left(\frac{\sqrt{(18{,}200.930 - 18{,}061.311)^2 + (56{,}476.686 - 56{,}042.621)^2}}{455.968} \right) - 1 = \left(\frac{455.967}{455.968} \right) - 1$$

= 0.9999978 – 1 ➔ *in parts per million* ➔ –0.0000022 × 1,000,000 = **–2.2 ppm**

Using Equation 3.2:

$$\delta = \frac{1.00025804 + 1.00025805}{2} \left(\frac{20{,}900{,}487}{20{,}900{,}487 + 5456} \right) - 1 = 0.9999970 - 1$$ ➔ **–3.0 ppm**

(= –0.016 ft/mile)

Exercise 3.2: Six steps for designing a low-distortion grid coordinate system

1. Define project area and choose *representative* ellipsoid height, h_o (not elevation)

- The *average* height of an area may not be appropriate (e.g., project near a mountain)
  - No need to estimate height to an accuracy of better than about ±20 feet
- Note that as the size of the area increases, the effect of Earth curvature on distortion increases and it must be considered in addition to the effect of topographic height
  - E.g., for areas wider than about 35 miles (perpendicular to the projection axis), distortion due to curvature alone exceeds 5 parts per million (ppm)

2. Place central meridian near centroid of project area

3. Scale central meridian of projection to representative ground height, h_o

Equation 3.3 Local map projection scaled to "ground"

$$\boxed{k_0 = 1 + \frac{h_0}{R_G}}$$

- Where R_G is geometric mean radius of curvature, $R_G = \dfrac{a\sqrt{1-e^2}}{1-e^2\sin^2\varphi}$ (Equation 1.7)
  - Alternatively, can initially approximate R_G as 20,900,000 feet for Arizona (since k_o will likely be refined in Step #4)
- This procedure is for the Transverse Mercator projection
  - For Lambert Conical projection, use same equation for scale of standard parallel

4. Check distortion at points distributed throughout project area

- Best approach is to compute distortion over entire area and generate distortion contours (this ensures optimal low-distortion coverage)
  - May require repeated evaluation using different k_o values
- Distortion computed at a point as $\delta = k\left(\dfrac{R_G}{R_G + h}\right) - 1$ (Equation 3.2)
  - Where k = projection grid scale factor at a point (with respect to ellipsoid; see Equations 3.3 and 3.4)
  - Multiply δ by 1,000,000 to get distortion in *parts per million* (ppm)

5. Keep the definition SIMPLE and CLEAN!

- Define k_o to no more than SIX decimal places, e.g., 1.000206 (exact)
  - *Note:* A change of one unit in the sixth decimal place equals distortion caused by a 21-foot change in height
- Defining central meridian and latitude of grid origin to nearest whole arc-minute usually adequate (e.g., Central meridian = 111°48'00" W)

- Define grid origin using large whole values with as few digits as possible (e.g., False easting = 50,000; Max coordinate < 100,000)

6. *Explicitly* define linear unit and geodetic datum

- E.g., Linear unit = International foot; Geodetic datum = NAD 83(2007)

Example computation
Design a Low Distortion Projection (LDP) for Prescott

1. Define project area and choose *representative* ellipsoid height, h_0 (<u>not</u> elevation)

From topographic maps and benchmark information, a representative ellipsoid height is h_0 = **<u>5400ft</u>** (no need for greater accuracy than nearest ±10 feet)

2. Place central meridian near centroid of project area

Based on location and extent of Prescott, a good, clean value is λ_0 = **<u>112° 28' 00" W</u>**

3. Scale central meridian of projection to representative ground height, h_0

First compute Earth radius at mid-latitude of Prescott, φ = 34° 32' 00" N (no need for greater accuracy than nearest arc-minute of latitude):

$$R_G = \frac{a\sqrt{1-e^2}}{1-e^2\sin^2\varphi} = \frac{20{,}925{,}646.325\times\sqrt{1-0.006694380023}}{1-0.006694380023\times[\sin(34.533333°)]^2} = \underline{20{,}900{,}450 \text{ ift}}$$

Thus the central meridian scale factor scaled to the representative ellipsoid height is

$$k_0 = 1+\frac{h_0}{R_G} = 1+\frac{5400}{20{,}900{,}450} = \mathbf{\underline{1.000258}}$$

Based on these results, the following Transverse Mercator projection is defined (will refine definition if necessary based on results of Step #4):

Latitude of grid origin,	φ_0 = 34° 30' 00" N
Longitude of central meridian,	λ_0 = 112° 28' 00" W
False northing,	N_0 = 0.000 ift
False easting,	E_0 = 50,000.000 ift
Central meridian scale factor,	k_0 = 1.000258

4. Check distortion at points distributed throughout project area

Distortion can be computed at various points throughout the project area. These can be survey control points or even artificial points taken from topographic maps.

To illustrate, we can use the results of the point distortion computation CAS-2 from the previous example (which is repeated here for convenience)

$$\delta = k\left(\frac{R_G}{R_G+h}\right)-1 = 1.00025804\times\left(\frac{20{,}900{,}487}{20{,}900{,}487+5467}\right)-1 = 0.999996468-1 = \mathbf{\underline{-3.5\text{ ppm}}}$$

For CAS-4 (at the hospital, 230 ft lower than the resort) we have:

$$\delta = k\left(\frac{R_G}{R_G + h}\right) - 1 = 1.00025802 \times \left(\frac{20{,}900{,}495}{20{,}900{,}495 + 5235}\right) - 1 = 1.000007546 - 1 = \underline{\mathbf{+7.5\ ppm}}$$

This computation can be performed at discrete points throughout the project area, but best approach is to compute distortion over entire area (for example on a 3-arc-second grid) and generate distortion contours to ensure optimal low-distortion coverage.

The ability to achieve low distortion is limited by change in elevation (height) within the project area. A reasonable goal might be to limit distortion to ±0.1 ft per mile, which is about ±20 ppm and corresponds to a height change of about ±400 ft.

5. Keep the definition SIMPLE and CLEAN!

All of the projection parameters were initially defined in Step #3, but trial-and-error may be necessary to refine definition.

- Note k_0 is defined to *exactly* SIX decimal places: $\boldsymbol{k_0}$ **= 1.000258 (exact)**

- Both latitude of grid origin and central meridian are defined to nearest whole arc-minute:

  $\boldsymbol{\varphi_0}$ **= 34° 30’ 00” N** and $\boldsymbol{\lambda_0}$ **= 112° 28’ 00” W**

  φ_0 was selected far enough south to ensure positive northings, but far enough north to keep northings less than 100,000 ift.

- Grid origin is defined using clean whole values with as few digits as possible:

  $\boldsymbol{N_0}$ **= 0.000 ift** and $\boldsymbol{E_0}$ **= 50,000.000 ift**

  These values were selected to keep grid coordinates positive but less than 100,000 ift within the Prescott area (it is conventional to set N_0 to zero at φ_0, but is not required).

6. *Explicitly* define linear unit and geodetic datum

Linear unit is **International foot**, and geodetic datum is **NAD 83(2007)**

Final Low Distortion Projection definition for this example:

Linear unit: International foot

Geodetic datum: North American Datum of 1983(2007)

System: Arizona LDP

Zone: Prescott

Projection: Transverse Mercator

Latitude of grid origin: 34° 30’ 00” N
Longitude of central meridian: 112° 28’ 00” W
Northing at grid origin: 0.000 ft
Easting at central meridian: 50,000.000 ft
Scale factor on central meridian: 1.000258 (exact)

Note that this coordinate system definition only deals with horizontal coordinates (no vertical datum is specified).

Methods for creating low-distortion grid coordinate systems

1. Design a Low Distortion Projection (LDP) for a specific project geographic area.

Use a conformal projection referenced to the existing geodetic datum.

Described in detail previously in this document.

2. Scale the reference ellipsoid "to ground".

A map projection referenced to this new "datum" is then designed for the project area.

Problems:

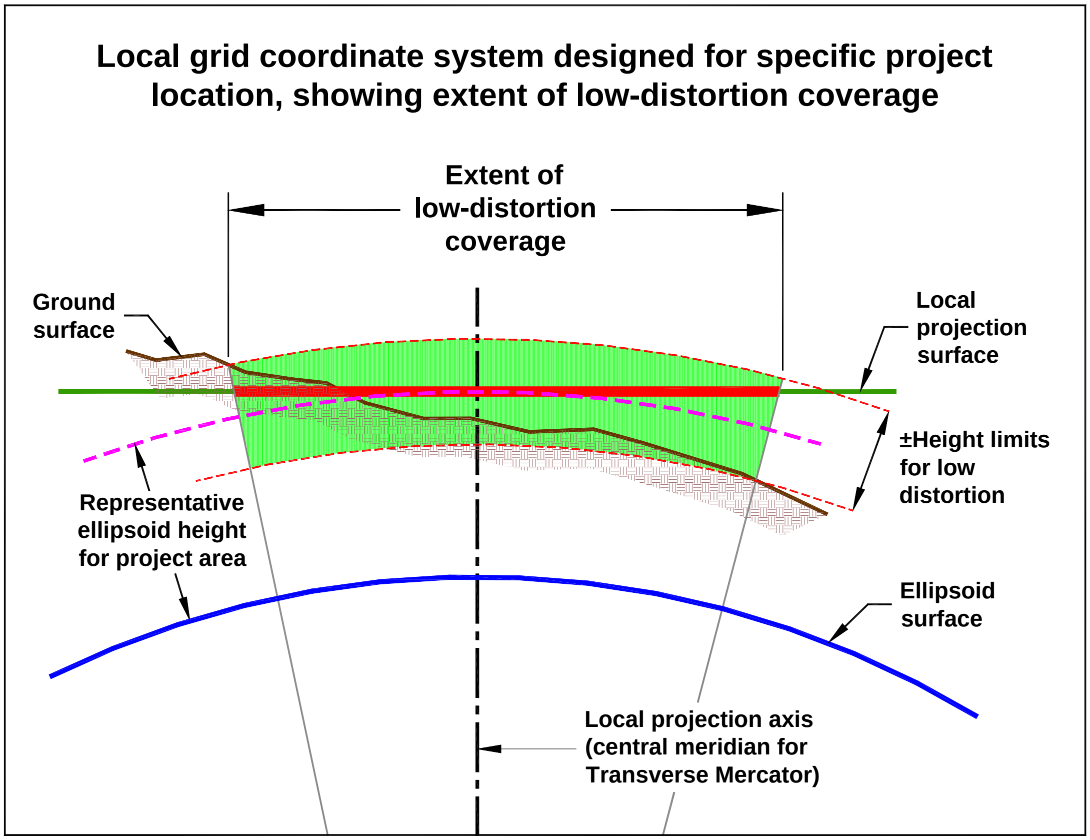
- Requires a new ellipsoid (datum) for every coordinate system, which makes it more difficult to implement than an LDP.
- New datum makes it more complex than an LDP, yet it does not perform any better.
- *Generates new set of latitudes that can be substantially different from original latitudes.*
    - Change in latitude can exceed 3 feet per 1000 ft of topographic height, depending on method used for scaling the ellipsoid (this case is for scaling with constant flattening).
    - Can lead to confusion over which latitude values are correct.

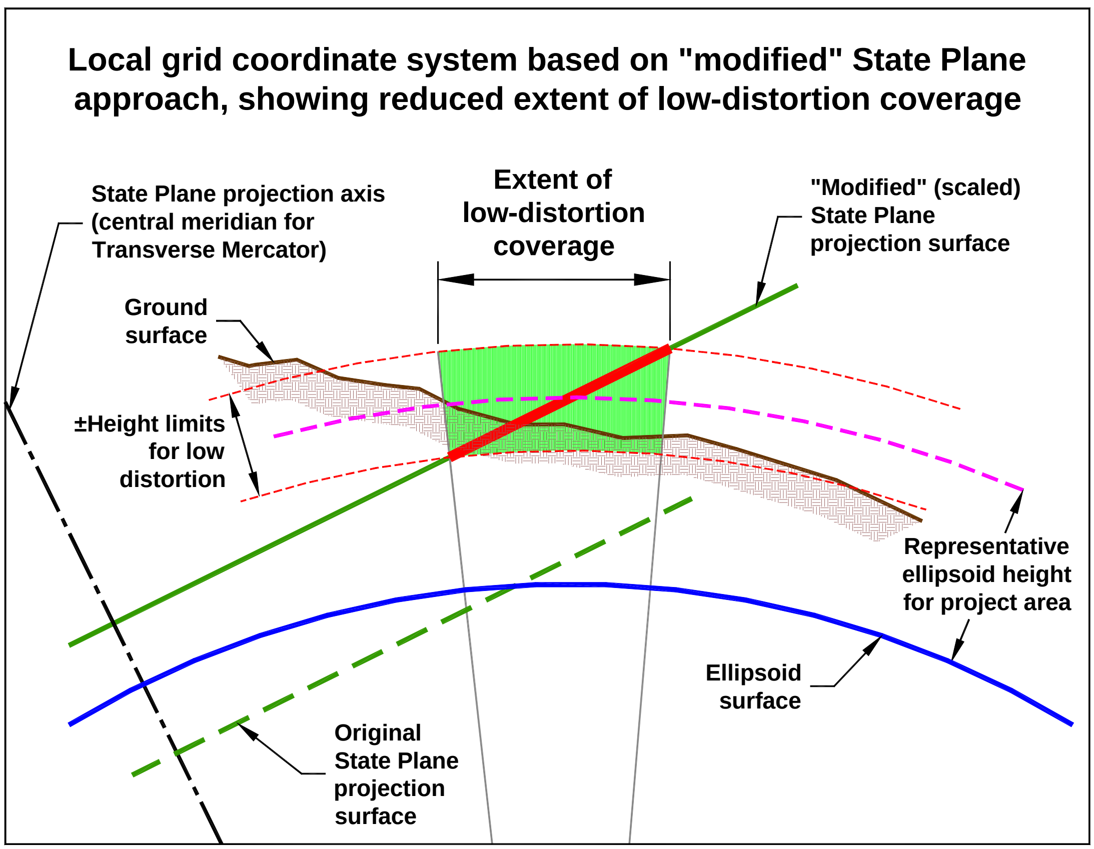
3. Scale an existing published map projection "to ground".

Referred to as "modified" State Plane when an existing SPCS projection definition is used.

Problems:

- Generates coordinates with values similar to "true" State Plane (can cause confusion).
    - Can eliminate this problem by translating grid coordinates to get smaller values.
- Often yields "messy" parameters when a projection definition is back-calculated from the scaled coordinates (in order to import the data into a GIS).
    - More difficult to implement in a GIS, and may cause problems due to rounding or truncating of "messy" projection parameters (especially for large coordinate values).
    - Can reduce this problem through judicious selection of "scaling" parameters.
- Does ***not*** reduce the convergence angle (it is same as that of original SPCS definition).
    - In addition, the *arc-to-chord correction* may be significant; it can reach ½ arc-second for a 1-mile line located 75 miles from the projection axis (this correction is used along with the convergence angle for converting grid azimuths to geodetic azimuths).
- ***MOST IMPORTANT:* Usually does not minimize distortion over as large an area as the other two methods.**
    - Extent of low-distortion coverage generally *decreases* as distance *increases* from projection axis (i.e., central meridian for TM and central parallel for LCC projection).
    - State Plane axis usually does NOT pass through the project area.
    - *Sketches illustrating this problem with "modified" SPCS are shown on the next page.*


Local grid coordinate system designed for specific project location, showing extent of low-distortion coverage
Extent of low-distortion coverage
Ground surface
Local projection surface
±Height limits for low distortion
Representative ellipsoid height for project area
Ellipsoid surface
Local projection axis (central meridian for Transverse Mercator)


Local grid coordinate system based on "modified" State Plane approach, showing reduced extent of low-distortion coverage
State Plane projection axis (central meridian for Transverse Mercator)
Extent of low-distortion coverage
"Modified" (scaled) State Plane projection surface
Ground surface
±Height limits for low distortion
Representative ellipsoid height for project area
Ellipsoid surface
Original State Plane projection surface

Exercise 3.3: Two methods for computing horizontal "ground" distance

This exercise gives two simple methods for computing horizontal "ground" distances using geodetic information. The first method is done by scaling the ellipsoid distance (geodesic) using the average of the ellipsoid heights at the endpoints, as follows:

Equation 3.4 Approximate geodetic "ground" distance based on ellipsoid distance (geodesic)

$$\boxed{D_{grnd} = s\left(1 + \frac{\bar{h}}{\bar{R}_G}\right)}$$

where s is the ellipsoid distance (geodesic)

$\bar{h}$ is the average ellipsoid height of the two points

$\bar{R}_G$ is the geometric mean radius of curvature at the midpoint latitude of the two points

The second method for computing a horizontal ground distance can be done by using a GPS (GNSS) vector directly. Neglecting Earth curvature, this distance can be computed as:

Equation 3.5 Approximate "ground" distance based on GPS (GNSS) vector components

$$\boxed{D_{grnd} = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2 - \Delta h^2}}$$

where $\Delta X, \Delta Y, \Delta Z$ are the GPS vector components (as ECEF Cartesian coordinate deltas)

Δh = change in ellipsoid height between vector end points

Note that this method can also be used with end point coordinates (rather than a GPS vector), by converting the latitude, longitude, and ellipsoid heights to X, Y, Z ECEF coordinates using Equation 2.1, and then using the difference in ECEF coordinates in Equation 3.5.

Accounting for curvature increases this horizontal ground distance, but for distances of less than 20 miles (about 30 km), the increase is less than 1 part per million (ppm), i.e., less than 0.1 ft (3 cm). The horizontal distance can be multiplied by the following curvature correction factor to get the approximate curved horizontal ground distance (error is less than about 0.01 ft for distances under 50 miles):

Equation 3.6 Correction factor applied to horizontal distance to account for curvature

$$\boxed{C_C = \frac{2\bar{R}_G \sin^{-1}\left(\dfrac{D_{grnd}}{2\bar{R}_G}\right)}{D_{grnd}}}$$

where all variables are as defined previously. An Earth radius of 20,900,000 ft is sufficiently accurate in Arizona for distances of less than about 100 miles (causes less than 0.01 ft error).

Example computation

Given: Points CAS-2 and CAS-3 from the previous exercises, and a GPS vector from CAS-2 to CAS-3 with components $\Delta X = 438.001$ ft, $\Delta Y = -76.678$ ft, and $\Delta Z = 103.056$ ft.

Find: The horizontal "ground" distance between these points using the two methods in this exercise.

Computations:

Method 1. From Exercises 2.3 and 2.4, ellipsoid distance (geodesic) is s = 455.849 ift

From Exercises 1.3 and 3.1, R_G = 20,900,487 ift at midpoint between CAS-2 and CAS-3 (which is the same as the average R_G for the two points)

From the ellipsoid heights in Exercise 3.1, the average ellipsoid height is

$$\bar{h} = (\_\_\_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_\_\_)/2 = \_\_\_\_\_\_\_\_\_\_ \text{ ift}$$

So ground distance is

$$D_{grnd} = s\left(1+\frac{\bar{h}}{\bar{R}_G}\right) = \_\_\_\_\_\_\_\_\_\_ \times \left(1+\frac{\quad}{\quad}\right) = \underline{\_\_\_\_\_\_\_\_\_\_ \textbf{ ift}}$$

Method 2. Using the given GPS vector components and Δh from Exercises 3.1 gives a horizontal ground distance of

$$D_{grnd} = \sqrt{(\_\_\_\_)^2 + (\_\_\_\_)^2 + (\_\_\_\_)^2 - (\_\_\_\_)^2} = \underline{\_\_\_\_\_\_\_\_\_\_ \textbf{ ift}}$$

Solution:

Method 1. From Exercise 2.4, ellipsoid distance (geodesic) is s = 455.849 ift

From Exercises 1.3 and 3.1, R_G = 20,900,487 ift at midpoint between CAS-2 and CAS-3 (which is the same as the average R_G for the two points)

From the ellipsoid heights in Exercise 3.1, the average ellipsoid height is

$$\bar{h} = (5466.883 + 5445.959) / 2 = 5456.421 \text{ ift}$$

So ground distance is

$$D_{grnd} = 455.849 \times \left(1+\frac{5456.421}{20{,}900{,}487}\right) = 455.849 \times 1.00026107 = \underline{\textbf{455.968 ift}}$$

Method 2. Using the given GPS vector components and Δh = 5445.959 – 5466.883 = −20.924 ft gives a horizontal ground distance of

$$D_{grnd} = \sqrt{(438.001)^2 + (-76.678)^2 + (103.056)^2 - (-20.924)^2} = \underline{\textbf{455.968 ift}},$$

which is the same as that computed for Method 1. For such a short distance, curvature is completely negligible. This can be verified using Equation 3.6, which gives a curvature correction factor of C_C = 1.000 000 000 020, or 0.00002 ppm. As stated previously, the curvature correction factor is less than 1 ppm for distances of less than about 20 miles.

Exercise 3.4: Projection grid scale factor and convergence angle computation

For the ***Transverse Mercator*** projection, the grid scale factor at a point can be computed as follows (modified from Stem, 1990, pp. 32-35):

Equation 3.7 Transverse Mercator projection grid scale factor formula

$$k = k_0 \left\{ 1 + \frac{(\Delta\lambda \cos\varphi)^2}{2} \left(1 + \frac{e^2 \cos^2\varphi}{1-e^2} \right) \left[1 + \frac{(\Delta\lambda \cos\varphi)^2}{12} \left(5 - 4\tan^2\varphi + \frac{e^2 \cos^2\varphi}{1-e^2} \left(9 - 24\tan^2\varphi \right) \right) \right] \right\}$$

where $\Delta\lambda = \lambda_0 - \lambda$ (in radians, for negative west longitude)

λ = geodetic longitude of point

λ_0 = central meridian longitude

and all other variables are as defined previously.

The following shorter equation can be used to approximate k for the Transverse Mercator projection. It is accurate to better than 0.02 part per million (at least 7 decimal places) if the computation point is within about ±1° of the central meridian (about 50 to 60 miles between latitudes of 30° and 45°):

Equation 3.8 Approximate Transverse Mercator projection grid scale factor formula

$$k \approx k_0 \left\{ 1 + \frac{(\Delta\lambda \cos\varphi)^2}{2} \left(1 + \frac{e^2 \cos^2\varphi}{1-e^2} \right) \right\}$$

Note that this equation may not be sufficiently accurate for computing k throughout a UTM system zone (at the zone width of ±3° from the central meridian the error can exceed 1 ppm).

An even simpler equation can be used to approximate the grid scale factor, which utilizes the grid coordinate easting value and is about twice as accurate as the previous equation (i.e., better than 0.01 part per million if the computation point is within about ±1° of the central meridian):

Equation 3.9 Another approximate Transverse Mercator projection grid scale factor formula

$$k \approx k_0 + \frac{(E_0 - E)^2}{2(k_0 R_G)^2}$$

where E = Easting of the point where k is computed (in same units as R_G)

E_0 = False easting (on central meridian) of projection definition (in same units as R_G)

R_G = Earth geometric mean radius of curvature (can use 20,900,000 feet for Arizona)

For the ***Lambert Conformal Conic*** projection, the grid scale factor at a point can be computed as follows (modified from Stem, 1990, pp. 26-29):

Equation 3.10 Lambert Conformal Conic projection grid scale factor formula

$$k = k_0 \frac{\cos\varphi_0}{\cos\varphi}\sqrt{\frac{1-e^2\sin^2\varphi}{1-e^2\sin^2\varphi_0}}\ \exp\left\{\frac{\sin\varphi_0}{2}\left[\ln\frac{1+\sin\varphi_0}{1-\sin\varphi_0} - \ln\frac{1+\sin\varphi}{1-\sin\varphi} + e\left(\ln\frac{1+e\sin\varphi}{1-e\sin\varphi} - \ln\frac{1+e\sin\varphi_0}{1-e\sin\varphi_0}\right)\right]\right\}$$

where k_0 = projection grid scale factor applied to central parallel (tangent to ellipsoid if k_0 = 1)

φ_0 = geodetic latitude of central parallel = standard parallel for one-parallel LCC

$e = \sqrt{e^2} = \sqrt{2f - f^2}$ = first eccentricity of the reference ellipsoid

and all other variables are as defined previously. In order to use this equation for a two-parallel LCC, the two-parallel LCC must first be converted to an equivalent one-parallel LCC by computing φ_0 and k_0. The equations to do this are long, but are provided here for the sake of completeness. For a two-parallel LCC, the central parallel is

$$\varphi_0 = \sin^{-1}\left[\frac{2\ln\left(\dfrac{\cos\varphi_S}{\cos\varphi_N}\sqrt{\dfrac{1-e^2\sin^2\varphi_N}{1-e^2\sin^2\varphi_S}}\right)}{\ln\left(\dfrac{1+\sin\varphi_N}{1-\sin\varphi_N}\right) - \ln\left(\dfrac{1+\sin\varphi_S}{1-\sin\varphi_S}\right) + e\left[\ln\left(\dfrac{1+e\sin\varphi_S}{1-e\sin\varphi_S}\right) - \ln\left(\dfrac{1+e\sin\varphi_N}{1-e\sin\varphi_N}\right)\right]}\right],$$

and the central parallel scale factor is

$$k_0 = \frac{\cos\varphi_N}{\cos\varphi_0}\sqrt{\frac{1-e^2\sin^2\varphi_0}{1-e^2\sin^2\varphi_N}}$$

$$\times\ \exp\left\{\frac{\sin\varphi_0}{2}\left[\ln\left(\frac{1+\sin\varphi_N}{1-\sin\varphi_N}\right) - \ln\left(\frac{1+\sin\varphi_0}{1-\sin\varphi_0}\right) + e\left(\ln\left[\frac{1+e\sin\varphi_0}{1-e\sin\varphi_0}\right] - \ln\left[\frac{1+e\sin\varphi_N}{1-e\sin\varphi_N}\right]\right)\right]\right\},$$

where φ_N and φ_S = geodetic latitude of northern and southern standard parallels, respectively, and all other variables are as defined previously.

Convergence angles. For the TM, the convergence angle can be approximated as $\gamma = -\Delta\lambda\sin\varphi$ (where all variables are as defined previously; the units of γ are the same as the units of $\Delta\lambda$). This equation is accurate to better than ±00.2" if the computation point is within ~1° of the central meridian. For any LCC, the convergence angle is exactly $\gamma = -\Delta\lambda\sin\varphi_0$.

Exercise 3.5: Grid versus geodetic bearings

Illustrates misclosure problem with geodetic azimuths, and shows how to convert grid azimuths to geodetic azimuths.

Equation 3.11 Relationship between grid and forward geodetic azimuth from point *A* to *B*

$$\boxed{\alpha_{AB} = t_{AB} + \gamma_A - (t - T)_{AB}}$$

where α_{AB} and t_{AB} = geodetic and grid azimuths from point *A* to *B*, respectively

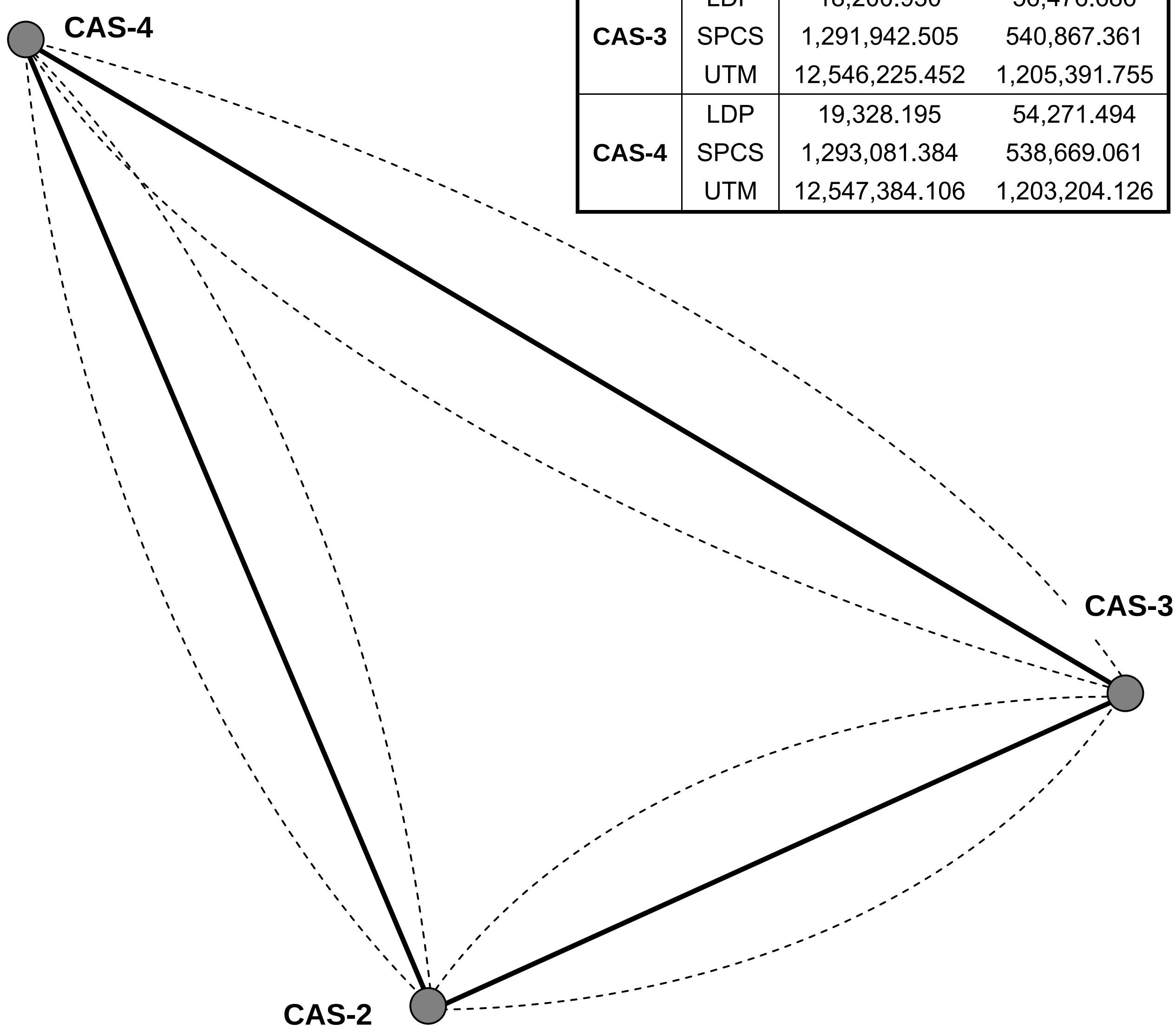
γ_A = map projection convergence angle at point *A*

$(t - T)_{AB}$ = Arc-to-chord ("second term") correction from *A* to *B* (usually negligible)

Example using Low Distortion Projection (LDP), State Plane, and UTM coordinates

Consider closed polygon below from points CAS-2 to CAS-3 to CAS-4 to CAS-2 (not to scale). Label the figure with distances, grid azimuths, and geodetic forward and back azimuths.

Grid coords		Northing (ift)	Easting (ift)
CAS-2	LDP	18,061.311	56,042.621
	SPCS	1,291,805.295	540,432.685
	UTM	12,546,092.208	1,204,955.902
CAS-3	LDP	18,200.930	56,476.686
	SPCS	1,291,942.505	540,867.361
	UTM	12,546,225.452	1,205,391.755
CAS-4	LDP	19,328.195	54,271.494
	SPCS	1,293,081.384	538,669.061
	UTM	12,547,384.106	1,203,204.126

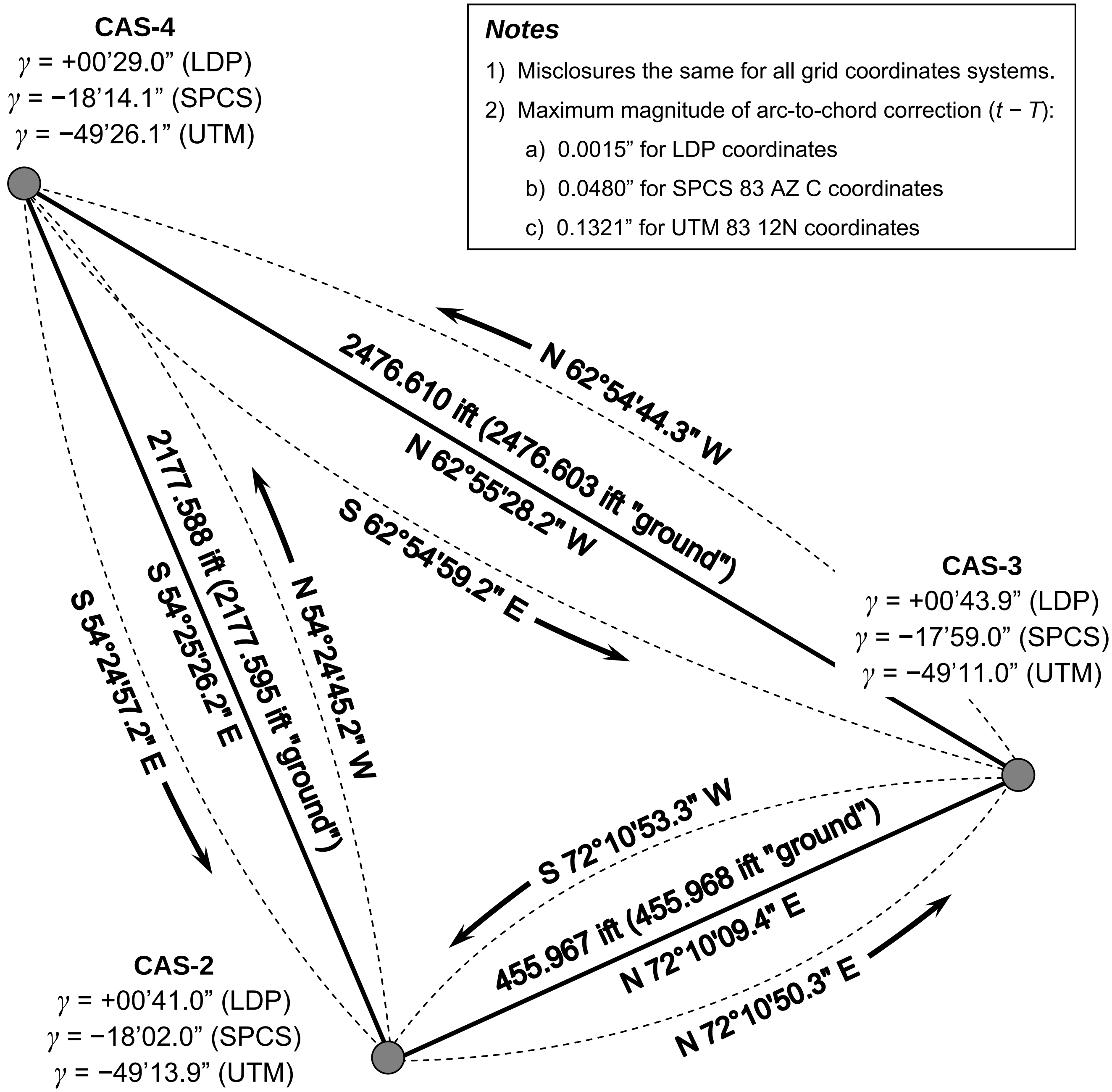


Example solution: Computed using Low Distortion Projection (LDP) coordinates

MISCLOSURES (computed using LDP coordinates)	
Grid bearings and grid distances (misclosure due to rounding)	0.0007 ft
Grid bearings and "ground" distances	0.0152 ft
Forward geodetic bearings and grid distances	0.1617 ft
Forward geodetic bearings and "ground" distances	0.1638 ft
Back geodetic bearings and grid distances	0.1484 ft
Back geodetic bearings and "ground" distances	0.1499 ft
Mean forward & back geodetic bearings and grid distances	0.0136 ft
Mean forward & back geodetic bearings and "ground" distances	0.0273 ft

Notes

1) Misclosures the same for all grid coordinates systems.
2) Maximum magnitude of arc-to-chord correction ($t - T$):
   a) 0.0015" for LDP coordinates
   b) 0.0480" for SPCS 83 AZ C coordinates
   c) 0.1321" for UTM 83 12N coordinates



Section 4

VERTICAL DATUMS AND HEIGHT SYSTEMS

How high is it? How deep is it? Where will water go?

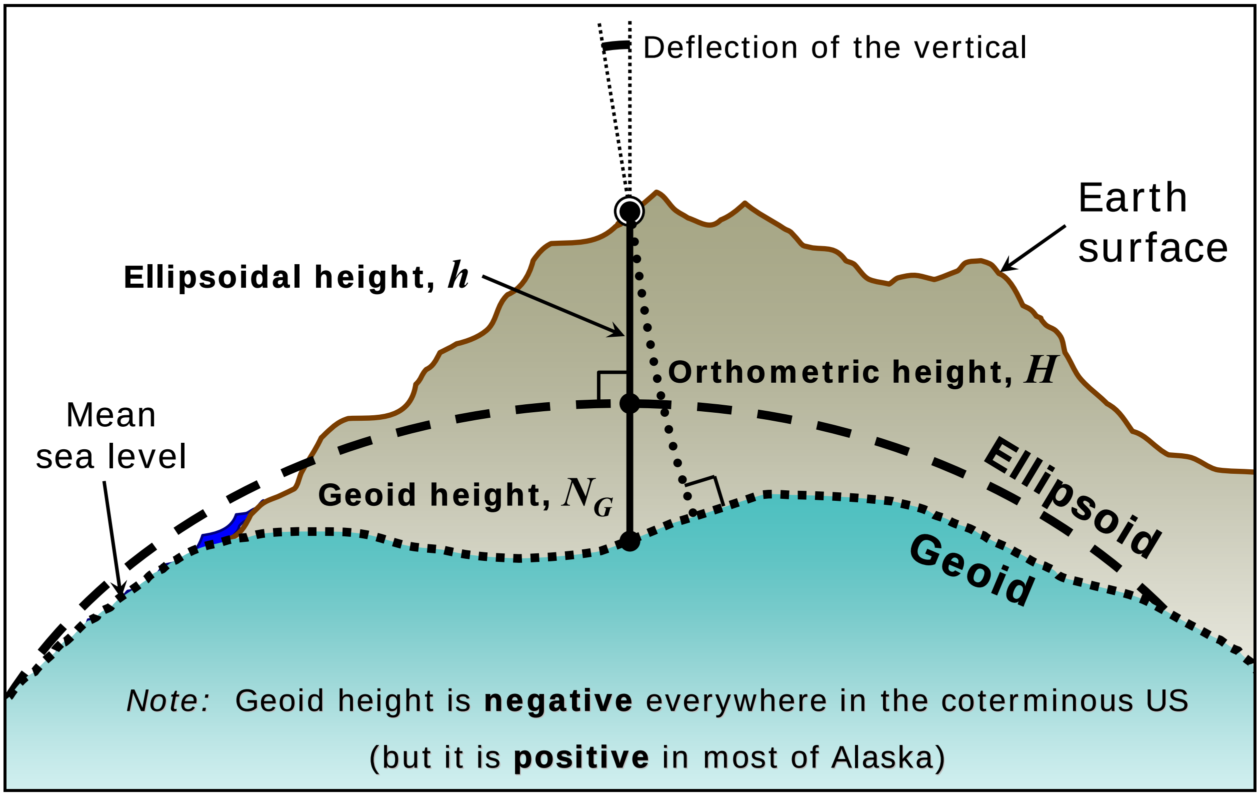
Examples of height determination errors for Arizona

Table 4.1 Examples of various positioning error sources and their magnitudes for Arizona due to vertical datum and height system problems (abbreviations and technical terms are defined in the Glossary).

Positioning error examples for Arizona	Error magnitudes
Using NGVD 29 when NAVD 88 required	1.2 to 4.5 feet (vertical)
Using ellipsoid heights for elevations	Varies from 63 feet to 113 feet (vertical)
Neglecting geoid slope when transferring elevations with GPS	Up to 0.7 foot vertical per mile horizontal
Using geoid model GEOID03 when GEOID09 is required to derive elevations from ellipsoid heights	Varies from −0.7 foot to +0.5 foot (vertical)
Using leveling without orthometric corrections to “correct” GPS-derived elevations	Can exceed 0.05 foot vertical per mile horizontal
Generating GPS-derived elevations using a best-fit inclined planar correction surface based on ties to inappropriate or inconsistent vertical control (via a vertical “calibration” or “localization”)	Varies, but can cause very large systematic vertical errors (can exceed several feet)

Exercise 4.1: Ellipsoid, orthometric, and geoid heights

The relationship between ellipsoidal, orthometric, and geoid heights is shown in the figure below. Note that everywhere in the coterminous US, the geoid height is negative (i.e., the geoid is below the ellipsoid). But in most of Alaska, the geoid height is positive.



Equation 4.1 Relationship between ellipsoidal, orthometric, and geoid heights

$$\boxed{h = H + N_G}$$

where h, H, and N_G are the ellipsoidal, orthometric, and geoid heights, respectively.

Strictly speaking, the relationship in Equation 4.1 is approximate due to deflection of the vertical. However, it is accurate at the sub-millimeter level, and so can be considered exact for all practical purposes.

Rules of Thumb:

Accuracy of NAVD 88 orthometric heights derived from NAD 83 ellipsoid heights using the following geoid models (based on NGS documentation and given at the 95% confidence level):

GEOID09 approximate absolute accuracy: ±0.09 ft (±2.7 cm)
GEOID03 approximate absolute accuracy: ±0.15 ft (±4.7 cm)
GEOID99 approximate absolute accuracy: ±0.30 ft (±9.0 cm)
GEOID96 approximate absolute accuracy: ±0.35 ft (±10.8 cm)

The relative accuracy of these geoid models is 1 to 2 ppm, or better.

Example computation

Given: An NGS Datasheet for conventional NGS control station PEND (below):

```
FQ0306 ***********************************************************************
FQ0306  DESIGNATION -  PEND
FQ0306  PID         -  FQ0306
FQ0306  STATE/COUNTY-  AZ/COCONINO
FQ0306  USGS QUAD   -  FLAGSTAFF WEST (1983)
FQ0306
FQ0306                         *CURRENT SURVEY CONTROL
FQ0306  ___________________________________________________________________
FQ0306* NAD 83(1992)-  35 11 18.46326(N)     111 41 28.38215(W)      ADJUSTED
FQ0306* NAVD 88     -      2160.187  (meters)     7087.21    (feet)  ADJUSTED
FQ0306  ___________________________________________________________________
FQ0306  LAPLACE CORR-          -2.27  (seconds)                      DEFLEC09
FQ0306  GEOID HEIGHT-         -23.11  (meters)                       GEOID09
FQ0306  DYNAMIC HT  -       2157.097 (meters)     7077.08   (feet)   COMP
FQ0306  MODELED GRAV-     979,125.4   (mgal)                         NAVD 88
FQ0306
FQ0306  HORZ ORDER  -  SECOND
FQ0306  VERT ORDER  -  FIRST     CLASS II
```

Find: The ellipsoid height of PEND in International and US Survey Feet.

Computations:

Sometimes the only horizontal control station available for a GPS survey was determined using conventional methods. These do not have an ellipsoid height, but there is enough information to compute it if an accurate NAVD 88 orthometric height is available. From the Datasheet we have:

$h = \quad H \quad + \quad N_G$

$h =$ ____________ + __________ = ______________m = ___________ift = ___________sft

Solution:

```
FQ0306 ***********************************************************************
FQ0306  DESIGNATION -  PEND
FQ0306  PID         -  FQ0306
FQ0306  STATE/COUNTY-  AZ/COCONINO
FQ0306  USGS QUAD   -  FLAGSTAFF WEST (1983)
FQ0306
FQ0306                         *CURRENT SURVEY CONTROL
FQ0306  ___________________________________________________________________
FQ0306* NAD 83(1992)-  35 11 18.46326(N)     111 41 28.38215(W)      ADJUSTED
FQ0306* NAVD 88     -      2160.187  (meters)     7087.21    (feet)  ADJUSTED   = H
FQ0306  ___________________________________________________________________
FQ0306  LAPLACE CORR-          -2.27  (seconds)                      DEFLEC09
FQ0306  GEOID HEIGHT-         -23.11  (meters)                       GEOID09    = N_G
FQ0306  DYNAMIC HT  -       2157.097 (meters)     7077.08   (feet)   COMP
FQ0306  MODELED GRAV-     979,125.4   (mgal)                         NAVD 88
FQ0306
FQ0306  HORZ ORDER  -  SECOND
FQ0306  VERT ORDER  -  FIRST     CLASS II
```

h = 2160.187 m + (−23.11 m) = <u>2137.08 m</u> = <u>7011.42 ift</u> = <u>7011.40 sft</u>

***h* = <u>7011.4 ft (±0.1 ft at 95% confidence)</u>** *in both International and US Survey Feet at accuracy of the computation*

Exercise 4.2: Trigonometric leveling

Equation 4.2 Change in elevation from trigonometric leveling

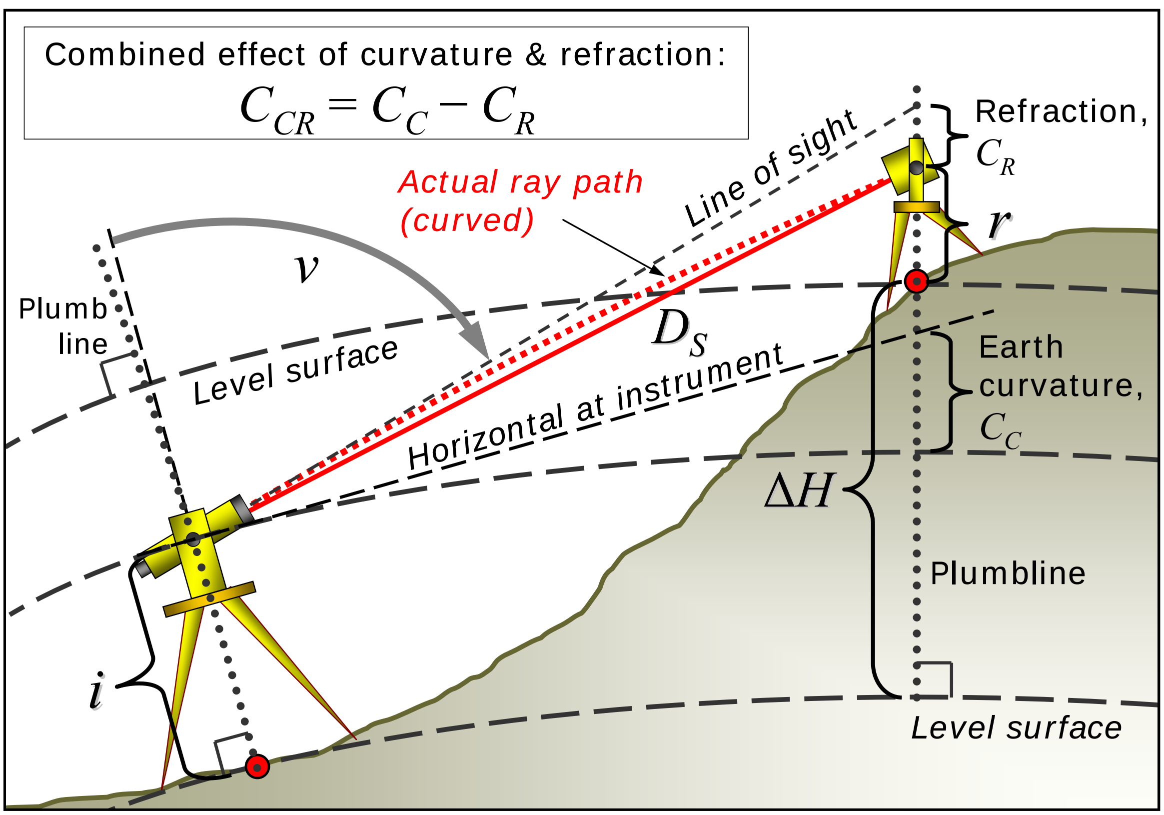
$$\boxed{\Delta H = D_S \cos\nu + i - r + C_{CR}}$$

where ΔH is the change in elevation (nominally orthometric height)
D_S is the slope distance
ν is the zenith angle
i and r are the instrument and prism rod heights, respectively, and
C_{CR} is the correction term for curvature and refraction, which is given by:

Equation 4.3 Curvature and refraction correction for trigonometric leveling (after Wolf and Brinker, 1994, p. 110)

$$\boxed{C_{CR} = 0.0206\left(\frac{D_S \sin\nu}{1000}\right)^2} \text{ [feet]} \qquad \boxed{C_{CR} = 0.0675\left(\frac{D_S \sin\nu}{1000}\right)^2} \text{ [meters]}$$

Note that C_{CR} is always *added* to the change in elevation computed in Equation 4.2.



Example computation

Given: Two survey stations (CAS-1 and CAS-4) were occupied with a 1-second Kern theodolite. The zenith angle was measured from both stations simultaneously in two sets of forward and reverse (face 1, face 2) observations, for a total of 12 measurements. The mean observed zenith angle for the forward and reverse sets are given below, along with the instrument and target heights. The slope distance between CAS-1 and CAS-4 is 2016.615 ift.

Sets	From	To	Instrument height (ft)	Target height (ft)	Mean zenith angle of all three sets
1-3	CAS-1	CAS-4	4.89	5.21	96° 37’ 39.0”
4-6	CAS-4	CAS-1	5.21	4.89	83° 22’ 41.3”

Find: The elevation change from CAS-1 to CAS-4, both with and without correction for curvature and refraction.

Computations:

The correction for curvature and refraction between these stations is essentially the same, since it is based on the horizontal distance, and so any of the observed zenith angles can be used:

$$C_{CR} = 0.0206\left(\frac{D_S \sin v}{1000}\right)^2 = 0.0206 \times \left(\frac{\underline{\qquad\qquad} \times \sin(\underline{\qquad\qquad\qquad})}{1000}\right)^2 = \underline{\qquad\qquad}\text{ ft}$$

Now compute each elevation change:

								Uncorrected			Corrected
	D_S	×	$\cos v$	+	i	−	r		+	C_{CR}	
ΔH_{1-3} =	________	× cos(	__________	) +	______	−	______	= _________	+	_______	= _________
ΔH_{1-3} =	________	× cos(	__________	) +	______	−	______	= _________	+	_______	= _________

<u>Average height change between stations:</u> _________ _________

Solution:

The correction for curvature and refraction between these stations is essentially the same, since it is based on the horizontal distance, and so any of the six zenith angles can be used:

$$C_{CR} = 0.0206 \times \left(\frac{2016.615 \times \sin(96.62750°)}{1000}\right)^2 = 0.0206 \times 2.0031^2 = \underline{0.083\text{ ft}}$$

Now compute each elevation change:

	Uncorrected		Corrected
$\Delta H_{1-3} = 2016.615 \times \cos(96.62750°) + 4.89 - 5.21 =$	$\underline{-233.065\text{ ft}}$	+ 0.083 =	$\underline{-232.982\text{ ft}}$
$\Delta H_{4-6} = 2016.615 \times \cos(83.37814°) + 5.21 - 4.89 =$	$\underline{+232.868\text{ ft}}$	+ 0.083 =	$\underline{+232.951\text{ ft}}$
<u>Average height change between stations:</u>	**<u>232.97 ft</u>**		**<u>232.97 ft</u>**

The average of uncorrected height changes is the same since effect of curvature and refraction cancels when observations are made in both directions simultaneously.

Exercise 4.3: Dynamic heights and geopotential numbers

In addition to orthometric heights, H ("elevations"), NGS Datasheets also give *dynamic heights*, H^D. A dynamic "height" is actually not a height in the geometric sense of a distance above a reference surface. Rather, it is a *geopotential number*, C, that has been divided (scaled) by a constant value of gravity, which gives H^D units of length. Both C and H^D represent the gravitational potential energy at a point, and changes in H^D are the only "height" differences that give true change in hydraulic head. That is, unconfined water will not flow from one point to another if the water surface at both points has the same H^D, even though the points will generally ***not*** have the same "elevation", H (i.e., $\Delta H^D \neq \Delta H$, although the difference is often small).

Equation 4.4 Relationship between dynamic height and geopotential number

$$\boxed{H^D = \frac{C}{\gamma_0}} \qquad \boxed{H^D = \frac{C}{9.806199}} \text{ [meters]} \qquad \boxed{H^D = \frac{C}{32.172569}} \text{ [ift]}$$

where C = geopotential number (units of m^2/s^2 or ft^2/s^2)

γ_0 = 9.806199 m/s^2 = normal gravity on the GRS 80 ellipsoid at 45° latitude (given on NGS Datasheets as 980.6199 gals, where 1 m/s^2 = 100 gals)

Both the dynamic and orthometric heights shown on NGS Datasheets were originally computed from the same set of adjusted geopotential numbers. The relationship between these two types of heights is given below.

Equation 4.5 Relationship between NAVD 88 dynamic and Helmert orthometric heights

$$\boxed{H^D = \frac{H}{\gamma_0}\bar{g} = \frac{H}{\gamma_0}\left(g + \frac{H}{K}\right) = \frac{H}{\gamma_0}\left(g + \frac{H}{2{,}358{,}000}\right)}$$

(modified from Zilkoski et al., 1992)

where $\bar{g}$ = Helmert mean gravity on the plumbline

g = "Observed" (modeled) NAVD 88 surface gravity (given on NGS Datasheets in milligals, where 1 m/s^2 = 100,000 mgals)

K = 2,358,000 s^2 = 1 / (4.24 × 10^{-7} s^{-2}) is a constant factor for computing Helmert NAVD 88 mean gravity (assumes constant topographic density of 2670 kg/m^3)

Equations 4.4 and 4.5 show that orthometric heights can also be computed from geopotential numbers, as $H = C / \bar{g}$.

Example computation

Given: The NGS Datasheet for NGS station PEND (in Exercise 4.1, and on the next page):

Find: The geopotential number of PEND from both the dynamic and orthometric height (in ift).

Computations:

Using the published NAVD 88 dynamic height:

$$C = \gamma_0 \times H^D$$

$$C = \underline{\qquad\qquad\qquad} \times \frac{\underline{\qquad\qquad\qquad}\ \text{m}}{0.3048\ \text{m/ift}} = \underline{\qquad\qquad\qquad}\ \text{ift}^2/\text{s}^2$$

Using the published NAVD 88 Helmert orthometric height:

$$C = \left(g + \frac{H}{K} \right) \times H$$

$$C = \left(\underline{\qquad\qquad} + \frac{\underline{\qquad\qquad}}{\underline{\qquad\qquad}} \right) \times \underline{\qquad\qquad\qquad} = \frac{\underline{\qquad\qquad\qquad}\ \text{m}^2/\text{s}^2}{(0.3048)^2\ \text{m}^2/\text{ift}^2} =$$

$$= \underline{\qquad\qquad\qquad}\ \text{ift}^2/\text{s}^2$$

```
FQ0306 ***********************************************************************
FQ0306  DESIGNATION -  PEND
FQ0306  PID         -  FQ0306
FQ0306  STATE/COUNTY-  AZ/COCONINO
FQ0306  USGS QUAD   -  FLAGSTAFF WEST (1983)
FQ0306
FQ0306                          *CURRENT SURVEY CONTROL
FQ0306  ___________________________________________________________________
FQ0306* NAD 83(1992)-  35 11 18.46326(N)     111 41 28.38215(W)     ADJUSTED
FQ0306* NAVD 88     -      2160.187  (meters)    7087.21   (feet)  ADJUSTED    = H
FQ0306  ___________________________________________________________________
FQ0306  LAPLACE CORR-         -2.27  (seconds)                     DEFLEC09
FQ0306  GEOID HEIGHT-        -23.11  (meters)                      GEOID09
FQ0306  DYNAMIC HT  -      2157.097 (meters)     7077.08   (feet)  COMP        = H^D
FQ0306  MODELED GRAV-    979,125.4   (mgal)                        NAVD 88     = g
FQ0306
FQ0306  HORZ ORDER  -  SECOND
FQ0306  VERT ORDER  -  FIRST     CLASS II
  :
FQ0306.The dynamic height is computed by dividing the NAVD 88
FQ0306.geopotential number by the normal gravity value computed on the
FQ0306.Geodetic Reference System of 1980 (GRS 80) ellipsoid at 45
FQ0306.degrees latitude (g = 980.6199 gals.).                                  = γ0
FQ0306
FQ0306.The modeled gravity was interpolated from observed gravity values.
```

Solution:

Using the published NAVD 88 dynamic height:

$$C = 32.172569\ \text{ift/s}^2 \times \frac{2157.097\ \text{m}}{0.3048\ \text{m/ift}} = \mathbf{\underline{227{,}688}\ ift^2/s^2}$$

Using the published NAVD 88 Helmert orthometric height:

$$C = \left(9.791254\ \text{m/s}^2 + \frac{2160.187\ \text{m}}{2{,}358{,}000\ \text{s}^2} \right) \times 2160.187\ \text{m} = \frac{21{,}152.92\ \text{m}^2/\text{s}^2}{(0.3048)^2\ \text{m}^2/\text{ift}^2} = \mathbf{\underline{227{,}688}\ ift^2/s^2}$$

Exercise 4.4: Computing orthometric and dynamic heights from leveling

Leveling, by itself, does not yield true change in orthometric or dynamic heights. But when leveling is combined with surface gravity, the change in geopotential numbers can be computed. If the geopotential number is known for at least one point in a leveling network, then it can be computed at all points in the network. The geopotential numbers can then be converted to orthometric and dynamic heights using the relationships from the previous section, where orthometric height is $H = C / \overline{g}$, and dynamic height is $H^D = C / \gamma_0$.

Equation 4.6 Determining change in geopotential from leveled height differences

$$\boxed{C_B \approx C_A + \left(\frac{g_A + g_B}{2} \right) \Delta n_{AB}}$$

where g_A and g_B = surface gravity at adjacent stations *A* and *B* (in m/s^2 or ft/s^2)

Δn_{AB} = leveled height difference from station *A* and *B* (in same linear units as gravity)

Alternatively, leveled height differences can be converted to orthometric heights and dynamic heights by adding an orthometric correction (*OC*) or dynamic correction (*DC*) to observed leveled height differences between adjacent stations.

Equation 4.7 The NAVD 88 Helmert orthometric correction for leveled height differences

$$\boxed{OC_{AB} \approx \frac{\left[K\left(g_A - g_B\right) - 2\,\Delta n_{AB}\right]\left[2H_A + \Delta n_{AB}\right]}{2\left(K\,g_B + H_A + \Delta n_{AB}\right)}}$$

(modified from Hwang and Hsiao, 2003)

where all variables are as defined previously, and the orthometric correction is added to the observed leveled height difference, i.e., $H_B \approx H_A + \Delta n_{AB} + OC_{AB}$.

Equation 4.8 The dynamic correction for leveled height differences

$$\boxed{DC_{AB} \approx \left(\frac{g_A + g_B}{2\gamma_0} - 1 \right) \Delta n_{AB}}$$

(modified from Hofmann-Wellenhof and Moritz, 2005)

where all variables are as defined previously, and the dynamic correction is added to the observed leveled height difference, i.e., $H_B^D \approx H_A^D + \Delta n_{AB} + DC_{AB}$.

"Approximately equal" symbols were used for equations 4.6 – 4.8 because the surface gravity varies continuously along the leveling route. These equations will be exactly true only when the gravity varies linearly between stations. For best results they should be applied to every turning point on a leveling route. However, in most cases, Equation 4.7 (orthometric corrections) should work well for stations less than about 2 km apart. Equations 4.6 and 4.8 (geopotential numbers and dynamic corrections) are more sensitive to variation in surface gravity, and may not give good results even for stations less than 2 km apart, especially in mountainous areas.

Example computation

Given: A leveled height difference of +50.387 ft measured from NGS stations M 504 (PID FQ0543) to L 504 (PID FQ0544). The following data apply to these stations:

	M 504 (*station A*)	**L 504 (*station B*)**
Orthometric height	6104.396 ift	?
Dynamic height	6095.991 ift	?
Surface gravity	32.125673 ift/s^2	32.125305 ift/s^2

Find: The orthometric and dynamic heights of L 504 (in ift). The stations are 6450 ft apart.

Computations: The stations are (slightly) less than about 2 km apart, so using gravity values only at the stations themselves should be adequate (rather than at every leveling turning point).

Alternative 1: Solve using geopotential numbers.

$$C_B = C_A + \left(\frac{g_A + g_B}{2}\right)\Delta n_{AB} \quad = \quad \gamma_0\, H_A^D + \left(\frac{g_A + g_B}{2}\right)\Delta n_{AB}$$

$$C_B = \underline{\qquad\qquad} \times \underline{\qquad\qquad} + \left(\frac{\underline{\qquad\qquad} + \underline{\qquad\qquad}}{2}\right) \times \underline{\qquad\qquad\qquad}$$

$$C_B = \underline{\qquad\qquad\qquad\qquad} \text{ ift}^2/\text{s}^2$$

Orthometric height:

$$H_B = \frac{C_B}{\bar{g}_B} = \frac{C_B}{g_B + \dfrac{H_A + \Delta n_{AB}}{K}} = \frac{\underline{\qquad\qquad}}{\underline{\qquad\qquad} + \dfrac{\underline{\qquad\qquad} + \underline{\qquad\qquad}}{\underline{\qquad\qquad}}}$$

$$H_B = \underline{\qquad\qquad\qquad} \textbf{ ift}$$

Dynamic height:

$$H_B^D = \frac{C_B}{\gamma_0} = \frac{\underline{\qquad\qquad}}{\underline{\qquad\qquad}} = \underline{\qquad\qquad\qquad} \textbf{ ift}$$

Alternative 2: Solve using dynamic and orthometric corrections.

$$OC_{AB} = \frac{\left[K \times (g_A - g_B) - 2 \times \Delta n_{AB}\right] \times \left[2 \times H_A + \Delta n_{AB}\right]}{2 \times (K \times g_B + H_A + \Delta n_{AB})}$$

$$OC_{AB} = \frac{\left[\underline{\qquad\qquad} \times (\underline{\qquad\qquad} - \underline{\qquad\qquad}) - 2 \times \underline{\qquad\qquad}\right] \times \left[2 \times \underline{\qquad\qquad} + \underline{\qquad\qquad}\right]}{2 \times (\underline{\qquad\qquad} \times \underline{\qquad\qquad} + \underline{\qquad\qquad} + \underline{\qquad\qquad})}$$

$$OC_{AB} = \underline{\qquad\qquad\qquad} \text{ft}$$

$$DC_{AB} = \left(\frac{g_A + g_B}{2 \times \gamma_0} - 1\right) \times \Delta n_{AB}$$

$$DC_{AB} = \left(\frac{\underline{\qquad\qquad} + \underline{\qquad\qquad}}{2 \times \underline{\qquad\qquad}} - 1\right) \times \underline{\qquad\qquad\qquad} = \underline{\qquad\qquad\qquad} \text{ft}$$

Orthometric height:

$H_B \approx H_A + \Delta n_{AB} + OC_{AB} =$ ____________ + _________ + ______ = ____________ **ift**

Dynamic height:

$H_B^D \approx H_A^D + \Delta n_{AB} + DC_{AB} =$ ____________ + _________ + ______ = ____________ **ift**

Solution:

Alternative 1: Solve using geopotential numbers.

$$C_B = 32.172569 \times 6095.991 + \left(\frac{32.125673 + 32.125303}{2} \right) \times 50.387$$

$$C_B = 196{,}123.7 + 32.125489 \times 50.387 = \underline{197{,}742.4} \text{ ift}^2/\text{s}^2$$

Orthometric height: $H_B = \dfrac{C_B}{\bar{g}_B} = \dfrac{197{,}742.4}{32.125305 + \dfrac{6104.396 + 50.387}{2{,}358{,}000}} = \underline{\mathbf{6154.847}}$ **ift**

Dynamic height: $H_B^D = \dfrac{C_B}{\gamma_0} = \dfrac{197{,}742.4}{32.172569} = \underline{\mathbf{6146.304}}$ **ift**

Alternative 2: Solve using orthometric and dynamic corrections.

$$OC_{AB} = \frac{\left[2{,}358{,}000 \times (32.125673 - 32.125305) - 2 \times 50.387\right] \times \left[2 \times 6104.396 + 50.387\right]}{2 \times (2{,}358{,}000 \times 32.125305 + 6104.396 + 50.387)}$$

$$OC_{AB} = \frac{[\,766.970\,] \times [\,12{,}259.179\,]}{151{,}515{,}248} = \underline{+0.062} \text{ ft}$$

$$DC_{AB} = \left(\frac{32.125673 + 32.125305}{2 \times 32.172569} - 1 \right) \times 50.387 = (-0.001463) \times 50.387 = \underline{-0.074} \text{ ft}$$

Orthometric height:

$H_B = H_A + \Delta n_{AB} + OC_{AB} =$ 6104.396 + 50.387 + 0.062 = **<u>6154.845</u> ift**

Dynamic height:

$H_B^D = H_A^D + \Delta n_{AB} + DC_{AB} =$ 6095.991 + 50.387 + (−0.074) = **<u>6146.304</u> ift**

Check: The NGS Datasheet for station L 504 gives:

H_B = 1875.997 m = <u>6154.846 ift</u> and H_B^D = 1873.393 m = <u>6146.302 ift</u> ✓

These results are essentially equal to the NGS Datasheet values, to within the displayed precisions (±0.0005 m = ±0.0016 ft). However, part of the difference is likely also due to non-linear variation in gravity between the stations, which are 6450 ft (1.95 km) apart

Note that ΔH = 50.449 ft does *not* equal ΔH^D = 50.311 ft, and that only ΔH^D gives true change in hydraulic head (even though it is not really a change in "height", at least in the geometric sense).

Section 5

DOCUMENTATION AND ACCURACY REPORTING

Is it in the right place? By how much? How do you know?

Examples of documentation and accuracy reporting errors

Table 5.1 Examples of various positioning error sources and their magnitudes due to documentation and accuracy reporting problems (abbreviations and technical terms are defined in the Glossary).

Documentation error examples	Problem
Documenting geodetic datum as "WGS-84" when data actually referenced to NAD 83	Perpetuates confusion about "equivalence" of WGS-84 and NAD 83
Listing grid coordinates (such as SPCS) as "NAD 83"	NAD 83 is a geodetic datum, not a grid coordinate system
Documenting geodetic datum as "GRS-80"	GRS-80 is a reference ellipsoid, not a datum
Documenting vertical datum as "Mean Sea Level" (MSL)	There is no MSL datum in the US (name changed to NGVD 29 in 1976)
Using precision as an accuracy estimate with data containing systematic errors (e.g., incorrect reference coordinates)	Accuracy estimate is meaningless
Reporting horizontal error using unscaled standard deviation, rather than at the 95% confidence level (as specified by the FGDC)	Gives error estimates at 39% confidence level
Reporting vertical error using unscaled standard deviation, rather than at the 95% confidence level (as specified by the FGDC)	Gives error estimates at 68% confidence level
Using radial and circular estimates for horizontal error rather than semi-major axis of horizontal error ellipse	Typically makes errors appear less than actual
Using trivial vectors in GPS network adjustments	Varies, but always makes errors appear less than actual
Relying on precision computed by baseline processor for a single GPS vector as an indicator of accuracy	Varies, but precision value usually very optimistic and will not reveal systematic errors

Exercise 5.1: Computing error circle and ellipse from standard error components

Accuracies are given on the NGS Datasheet as linear values for the north, east, and up components (in centimeters) scaled to the 95% confidence level. The north and east components can be converted to a horizontal (circular) accuracy consistent with the approach used by the National Standard for Spatial Data Accuracy (NSSDA) as developed by the Federal Geographic Data Committee (1998, Part 3). Error ellipse axes and rotation can also be computed from the north and east standard error components and horizontal correlation given in the NGS Readjustment Distribution Format (RDF) file.

Equation 5.1 Horizontal (circular) accuracy computed from north and east accuracy components (at the 95% confidence level per NSSDA)

$$\boxed{CEP_{95} = 1.2489\ \frac{E_{95}^{N} + E_{95}^{E}}{2} = 2.4477\ \frac{\sigma_N + \sigma_E}{2}}$$

where CEP_{95} is the estimated Circular Error Probable (horizontal accuracy) at 95% confidence

E_{95}^{N} and E_{95}^{E} are the north and east errors (accuracies), respectively, from the NGS Datasheet (which are given at 95% confidence)

σ_N and σ_E are the north and east standard errors, respectively, from the NGS RDF file

The value 1.2489 is the ratio of the *bivariate* and *univariate* scalars for a confidence level of 95%, because the NGS Datasheet gives the north and east accuracies using the univariate scalar at 95% confidence (see Table 5.2 below for these scalars at this and other confidence levels). Note that *CEP* is typically computed at the 50% confidence level.

Equation 5.2 Horizontal covariance computed from correlation (the horizontal correlation is given in the NGS RDF file)

$$\boxed{\sigma_{NE} = \rho\, \sigma_N\, \sigma_E}$$

where σ_{NE} is the horizontal covariance

ρ is the horizontal correlation

Equation 5.3 Horizontal error ellipse axes computed from standard errors and covariance (at 95% confidence; standard error values are given in the NGS RDF file)

$$\boxed{a,\ b\ =\ 2.4477 \sqrt{\frac{1}{2}\left[\sigma_N^2 + \sigma_E^2\ \pm\ \sqrt{\left(\sigma_N^2 - \sigma_E^2\right)^2 + 4\sigma_{NE}^2}\right]}}$$

where *a* and *b* are the error ellipse semi-major and semi-minor axes, scaled to 95% confidence (note that the "±" operator allows computation of both *a* and *b* with this one equation, and that *a* is always greater than *b*).

Equation 5.4 Horizontal error ellipse rotation computed from standard errors and covariance (standard error values are given in the NGS RDF file)

$$\boxed{\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\sigma_{NE}}{\sigma_E^2 - \sigma_N^2}\right)}$$

where θ is the rotation angle of the semi-major axis, with respect to the east direction (positive counterclockwise). If $\sigma_N > \sigma_E$, rotation is with respect to the *positive* east axis. If $\sigma_N < \sigma_E$, rotation is with respect to the *negative* east axis. If $\sigma_N = \sigma_E$, then θ = ±45°, where the sign of the rotation is determined by the sign of σ_{NE}.

Table 5.2 Values used to scale standard errors (accuracies) to various confidence levels. The *univariate scalar* is used for single error components, such as vertical error. The *bivariate scalar* is used for dual (two-dimensional) error components, such as horizontal error, and can be used to scale an error ellipse to a desired confidence level. The *trivariate scalar* is rarely used but is provided here for the sake of completeness. It is for three-dimensional error components and can be used for scaling an error ellipsoid to a desired confidence level. In all cases, these scalars are based on the normal probability distribution of random variables, and the multivariate scalars are for jointly distributed random variables.

Univariate scalars		**Bivariate scalars**		**Trivariate scalars**	
Scalar, c_X^1	**Confidence level,** X	**Scalar,** c_X^2	**Confidence level,** X	**Scalar,** c_X^3	**Confidence level,** X
0.6745	50.00%	1.0000	39.35%	1.0000	19.87%
1.0000	68.27%	1.1774	50.00%	1.5382	50.00%
1.6449	90.00%	2.0000	86.47%	2.0000	73.85%
1.9600	95.00%	2.1460	90.00%	2.5003	90.00%
2.0000	95.45%	2.4477	95.00%	2.7955	95.00%
2.5758	99.00%	3.0000	98.89%	3.0000	97.07%
3.0000	99.73%	3.0349	99.00%	3.3682	99.00%
3.2905	99.90%	3.7169	99.90%	4.0331	99.90%

Again, for the sake of completeness, note that the trivariate scalar can be used to scale the estimated Spherical Error Probable (*SEP*) to a desired confidence level. The estimated *SEP* at 95% confidence is computed as

$$SEP_{95} = 2.7955\,\frac{\sigma_N + \sigma_E + \sigma_U}{3} = 1.4263\,\frac{E_{95}^N + E_{95}^E + E_{95}^U}{3},$$

where σ_U is the up (ellipsoid height) standard error and E_{95}^U is the up error (accuracy) at 95% confidence as given on the NGS Datasheet. As with *CEP*, typically *SEP* is computed at 50% confidence.

Example computation

Given: The NGS Datasheet network accuracies and Readjustment Distribution Format (RDF) standard errors (sigmas) for station NN 03 (CZ2412):

```
CZ2412 ***********************************************************************
CZ2412  DESIGNATION -  NN 03
CZ2412  PID         -  CZ2412
CZ2412  STATE/COUNTY-  AZ/PIMA
CZ2412  USGS QUAD   -  RUELAS CANYON (1992)
CZ2412
CZ2412                         *CURRENT SURVEY CONTROL
CZ2412  ___________________________________________________________________
CZ2412* NAD 83(2007)-  32 25 25.89923(N)    111 03 21.09845(W)     ADJUSTED
CZ2412* NAVD 88     -       812.8    (meters)    2667.     (feet)  GPS OBS
CZ2412  ___________________________________________________________________
 .
 .
CZ2412  ------- Accuracy Estimates (at 95% Confidence Level in cm) --------
CZ2412  Type    PID    Designation                      North   East  Ellip
CZ2412  -------------------------------------------------------------------
CZ2412  NETWORK CZ2412 NN 03                             1.18   1.20   2.16
CZ2412  -------------------------------------------------------------------
```

E_{95}^{N} E_{95}^{E} E_{95}^{U}

Readjustment Distribution Format values for station NN 03 (in centimeters):

```
000010*A1*HZTLOBS                                                 20060712
000020*10*az.bfile
000030*13*NAD 83(NSRS 2007)      GRS-80           6378137000          6356752314
 .
 .
CZ2412*80*0164NN 03                          32252589923N111032109845W       AZ
CZ2412*86*0164                                 783662
 .
 .
DH5794*91*1348             0.60       0.61-.18761247        1.10      Y
```

σ_N σ_E ρ σ_U

Find: The ellipsoid height (up) error, circular error probable (*CEP*), spherical error probable (*SEP*), and the horizontal error ellipse axes and rotation angle, all at the 50%, 95%, and 99% confidence levels. Compute all values from either or both the accuracy values on the datasheet or the RDF output, as appropriate, and give the final results in feet.

Computations: The ellipsoid height accuracy (error) is one-dimensional, so the univariate scalars from Table 5.2 should be used to scale the errors to the required confidence levels. This can be done from either the RDF file or the Datasheet.

Using the RDF sigma value *Using the Datasheet accuracy*

$$E_X^U = c_X^1 \times \sigma_U \qquad or \qquad E_X^U = \frac{c_X^1}{c_{95}^1} \times E_{95}^U$$

where c_X^1 is the univariate scalar at the X% confidence level, and we have:

$$E_{50}^{U} = \left\{ \begin{array}{c} 0.6745 \times 1.10 \\ or \\ 0.6745 / 1.9600 \times 2.16 \end{array} \right\} = 0.74 \text{ cm} = \underline{\mathbf{0.024\ ft\ (at\ 50\%\ confidence)}}$$

$$E_{95}^{U} = \left\{ \begin{array}{c} 1.9600 \times 1.10 \\ or \\ 1.9600 / 1.9600 \times 2.16 \end{array} \right\} = 2.16 \text{ cm} = \underline{\mathbf{0.071\ ft\ (at\ 95\%\ confidence)}}$$

$$E_{99}^{U} = \left\{ \begin{array}{c} 2.5758 \times 1.10 \\ or \\ 2.5758 / 1.9600 \times 2.16 \end{array} \right\} = 2.84 \text{ cm} = \underline{\mathbf{0.093\ ft\ (at\ 99\%\ confidence)}}$$

The *CEP* is two-dimensional, so the bivariate scalars from Table 5.2 should be used to scale the errors to the required confidence levels. This can also be done from either the RDF file or the Datasheet.

Using the RDF sigma values *Using the Datasheet accuracies*

$$CEP_X = c_X^2 \frac{\sigma_N + \sigma_E}{2} \qquad or \qquad CEP_X = \frac{c_X^2}{c_{95}^1} \times \frac{E_{95}^N + E_{95}^E}{2}$$

where c_X^2 is the bivariate scalar at the X% confidence level (note that the univariate scalar is used in the denominator for Datasheet accuracies). First we can compute the mean north and east sigma and Datasheet accuracy values as $(\sigma_N + \sigma_E)$ / 2 = (0.60 + 0.61) / 2 = 0.605 cm and ($E_{95}^N + E_{95}^E$) / 2 = (1.18 + 1.20) / 2 = 1.190 cm. The *CEP* for each case is then:

$$CEP_{50} = \left\{ \begin{array}{c} 1.1774 \times 0.605 \\ or \\ 1.1774 / 1.9600 \times 1.19 \end{array} \right\} = 0.71 \text{ cm} = \underline{\mathbf{0.023\ ft\ (at\ 50\%\ confidence)}}$$

$$CEP_{95} = \left\{ \begin{array}{c} 2.4477 \times 0.605 \\ or \\ 2.4477 / 1.9600 \times 1.19 \end{array} \right\} = 1.49 \text{ cm} = \underline{\mathbf{0.049\ ft\ (at\ 95\%\ confidence)}}$$

$$CEP_{99} = \left\{ \begin{array}{c} 3.0349 \times 0.605 \\ or \\ 3.0349 / 1.9600 \times 1.19 \end{array} \right\} = 1.84 \text{ cm} = \underline{\mathbf{0.060\ ft\ (at\ 99\%\ confidence)}}$$

The three-dimensional *SEP* is computed in a similar manner with the trivariate scalars:

Using the RDF sigma values $\quad$ *Using the Datasheet accuracies*

$$SEP_X = c_X^3 \, \frac{\sigma_N + \sigma_E + \sigma_U}{3} \qquad or \qquad SEP_X = \frac{c_X^3}{c_{95}^1} \times \frac{E_{95}^N + E_{95}^E + E_{95}^U}{3}.$$

where c_X^3 is the trivariate scalar at the X% confidence level. The mean north, east, and up sigma and Datasheet accuracy values are

$$(\sigma_N + \sigma_E + \sigma_U) / 3 = (0.60 + 0.61 + 1.10) / 3 = \underline{0.770 \text{ cm}}$$

$$(E_{95}^N + E_{95}^E + E_{95}^U) / 3 = (1.18 + 1.20 + 2.16) / 3 = \underline{1.51 \text{ cm}},$$

and we have

$$SEP_{50} = \left\{ \begin{array}{c} 1.5382 \times 0.770 \\ or \\ 1.5382 / 1.9600 \times 1.51 \end{array} \right\} = \quad 1.19 \text{ cm} \quad = \quad \underline{\textbf{0.039 ft (at 50\% confidence)}}$$

$$SEP_{95} = \left\{ \begin{array}{c} 2.7955 \times 0.770 \\ or \\ 2.7955 / 1.9600 \times 1.51 \end{array} \right\} = \quad 2.16 \text{ cm} \quad = \quad \underline{\textbf{0.071 ft (at 95\% confidence)}}$$

$$SEP_{99} = \left\{ \begin{array}{c} 3.3682 \times 0.770 \\ or \\ 3.3682 / 1.9600 \times 1.51 \end{array} \right\} = \quad 2.60 \text{ cm} \quad = \quad \underline{\textbf{0.085 ft (at 99\% confidence)}}$$

The horizontal error ellipse must be computed from the RDF values, because the horizontal correlation is not given on the datasheet. We can compute the horizontal covariance from the RDF correlation value:

$$\sigma_{NE} = \rho \, \sigma_N \, \sigma_E = -0.18761247 \times 0.60 \text{ cm} \times 0.61 \text{ cm} = \underline{-0.06867 \text{ cm}^2}.$$

The standard error ellipse axes can now be computed using Equation 5.3 (with the c_X^2 value set to one). Note that there is a "±" symbol in the equation — a is computed for the case where "±" is "+", and b is computed for the case where "±" is "–":

$$a, b = \sqrt{\frac{1}{2}\left[\sigma_N^2 + \sigma_E^2 \;\pm\; \sqrt{\left(\sigma_N^2 - \sigma_E^2\right)^2 + 4\sigma_{NE}^2}\right]}$$

$$= \sqrt{\frac{1}{2}\left[0.60^2 + 0.61^2 \;\pm\; \sqrt{\left(0.60^2 - 0.61^2\right)^2 + 4 \times \left(-0.06867\right)^2}\right]} = \left\{ \begin{array}{l} \underline{a = 0.66 \text{ cm} = 0.022 \text{ ft}} \\ \\ \underline{b = 0.55 \text{ cm} = 0.018 \text{ ft}} \end{array} \right.$$

Since c_X^2 = 1.0000 for the previous computations, the *a* and *b* dimensions are for the standard error ellipse, which has a confidence level of 39.35% (as shown in Table 5.2). This can be scaled to the required confidence levels using the appropriate bivariate scalars, as follows:

$$a_{50},\ b_{50} = 1.1774 \left\{ \begin{array}{lllll} \times\, 0.66\text{ cm} & = & 0.78\text{ cm} & = & \underline{\mathbf{0.025\ ft}} \\ \times\, 0.55\text{ cm} & = & 0.64\text{ cm} & = & \underline{\mathbf{0.021\ ft}} \end{array} \right\} \underline{\textbf{(at 50\% confidence)}}$$

$$a_{95},\ b_{95} = 2.4477 \left\{ \begin{array}{lllll} \times\, 0.66\text{ cm} & = & 1.61\text{ cm} & = & \underline{\mathbf{0.053\ ft}} \\ \times\, 0.55\text{ cm} & = & 1.33\text{ cm} & = & \underline{\mathbf{0.044\ ft}} \end{array} \right\} \underline{\textbf{(at 95\% confidence)}}$$

$$a_{99},\ b_{99} = 3.0349 \left\{ \begin{array}{lllll} \times\, 0.66\text{ cm} & = & 2.00\text{ cm} & = & \underline{\mathbf{0.066\ ft}} \\ \times\, 0.55\text{ cm} & = & 1.65\text{ cm} & = & \underline{\mathbf{0.054\ ft}} \end{array} \right\} \underline{\textbf{(at 99\% confidence)}}$$

The error ellipse rotation is computed using Equation 5.4, as follows:

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\sigma_{NE}}{\sigma_E^2 - \sigma_N^2}\right) = \frac{1}{2}\tan^{-1}\left(\frac{2\times(-0.06867)}{0.61^2 - 0.60^2}\right) = \underline{-0.74146\text{ radian}}$$

Converting this to degrees gives:

$$\theta = \frac{180°}{\pi} \times -0.74146 = \underline{\mathbf{-42.482°}}$$

The convention is that right-handed (counterclockwise) rotation is positive. So for this case, the error ellipse is rotated 42.482° clockwise from cardinal directions.

Note the apparent discrepancy in "horizontal" accuracy between what is given on the Datasheet and what was computed for CEP and the error ellipse axes at the 95% confidence level. The mean value from the Datasheet 0.86 cm, whereas CEP_{95} = 1.08 cm (which is the same as the mean of the error ellipse axes at 95% confidence). The reason these values differ is that the Datasheet values are for each individual north and east component, and so to scale these one-dimensional values to 95% confidence level requires a *univariate* scalar of 1.9600. The CEP and error ellipse represent the two-dimensional horizontal accuracies, which requires using the *bivariate* scalar of 2.4477 to scale these values to 95% confidence. Because of this, if you want to characterize the accuracies on the Datasheet as horizontal (rather than as individual cardinal components), then the Datasheet values should be scaled by the ratio 2.4477/1.9600 = 1.2489, as shown in Equation 5.1 and was done in the CEP_{95} computation in this example.

Surveying & mapping spatial data requirements & recommendations

These should be explicitly specified in surveying and mapping projects

1. Completely define the coordinate system

a. Linear unit (e.g., International foot, U.S. Survey foot, meter)
   i. Use same linear unit for horizontal and vertical coordinates
b. Geodetic datum (recommend North American Datum of 1983)
   i. Should include datum "tag" (date), e.g., 1986, 1992 (HARN), 2002.0 (CORS)
   ii. WGS 84, ITRF, and NAD 27 are **NOT** recommended
c. Vertical datum (e.g., North American Vertical Datum of 1988)
   i. If GPS used for elevations, recommend using a modern geoid model (e.g., GEOID03)
   ii. Recommend using NAVD 88 rather than NGVD 29 when possible
d. Map projection type and parameters (e.g., Transverse Mercator, Lambert Conformal Conic)
   i. Special attention required for low-distortion grid (a.k.a. "ground") coordinate systems
      1) Avoid scaling of existing coordinate systems (e.g., "modified" State Plane)

2. Require *direct* referencing of the NSRS (National Spatial Reference System)

a. Ties to published control strongly recommended (e.g., National Geodetic Survey control)
   i. Relevant component of control must have greater accuracy than positioning method used
      1) E.g., B-order (or better) stations for GPS control, 2nd order (or better) for vertical control
b. NGS Continuously Operating Reference Stations (CORS) can be used to reference the NSRS
   i. Free Internet GPS post-processing service: OPUS (Online Positioning User Service)

3. Specify *accuracy* requirements (*not* precision)

a. Use objective, defensible, and robust methods (published ones are recommended)
   i. Mapping and surveying: National Standard for Spatial Data Accuracy (NSSDA)
      1) Require occupations ("check shots") of known high-quality control stations
   ii. Surveys performed for establishing control or determining property boundaries:
      1) Appropriately constrained and over-determined least-squares adjusted control network
      2) Beware of "cheating" (e.g., using "trivial" GPS vectors in network adjustment)

4. Documentation is *essential* (metadata!)

a. Require a report detailing methods, procedures, and results for developing final deliverables
   i. This must include any and all post-survey coordinate transformations
      1) E.g., published datum transformations, computed correction surfaces, "rubber sheeting"
b. Documentation should be complete enough that someone else can reproduce the product
c. For GIS data, recommend that accuracy and coordinate system information be included as feature attributes (not just as separate, easy-to-lose and easy-to-ignore metadata files)

Example of surveying and mapping documentation (*metadata*)

<u>Basis of Bearings and Coordinates</u>

<u>Linear unit</u>: International foot (ift)

<u>Geodetic datum</u>: North American Datum of 1983 (2007)

<u>Vertical datum</u>: North American Vertical Datum of 1988 (see below)

<u>System</u>: Arizona LDP

<u>Zone</u>: Gila Valley

<u>Projection</u>: Transverse Mercator

Latitude of grid origin: 32° 20’ 00” N
Longitude of central meridian: 109° 48’ 00” W
Northing at grid origin: 0.000 ift
Easting at central meridian: 200,000.000 ift
Scale factor on central meridian: 1.00014 (exact)

All distances and bearings shown hereon are projected (grid) values based on the preceding projection definition. The projection was defined to minimize the difference between projected (grid) distances and horizontal (“ground”) distances at the topographic surface within the design area of this coordinate system.

The basis of bearings is geodetic north. Note that the grid bearings shown hereon (or implied by grid coordinates) do not equal geodetic bearings due to meridian convergence.

Orthometric heights (elevations) were transferred to the site from NGS control station “P 439” (PID CY0725) using GPS with NGS geoid model “GEOID09” referenced to the current published NAVD 88 height of this station (889.460 m).

The survey was conducted using GPS referenced to the National Spatial Reference System. A partial list of point coordinates is given below (additional coordinates are available upon request). Local network accuracy estimates are given at the 95% confidence level and are based on an appropriately constrained least-squares adjustment of over-determined and statistically independent observations.

<u>Point #1 “SAFFORD BASE ARP”, permanent GPS base (off site)</u>

Latitude = 32° 48' 07.31561” N	Northing = 170,563.997 ift	*<u>Estimated accuracy</u>*
Longitude = 109° 42' 42.84664” W	Easting = 227,075.294 ift	Horizontal = Fixed
Ellipsoidal height = 2945.423 ift	Elevation = 3033.826 ift	Vertical = Fixed

<u>Point #1002, 1/2” rebar with aluminum cap, derived coordinates (on site)</u>

Latitude = 32° 50' 06.81662" N	Northing = 182,643.211 ift	*<u>Estimated accuracy</u>*
Longitude = 109° 42' 47.90144" W	Easting = 226,633.861 ift	Horizontal = ±0.034 ift
Ellipsoidal height = 2822.412 ift	Elevation = 2910.734 ift	Vertical = ±0.056 ift

<u>Point #1006, 1/2” rebar with plastic cap, derived coordinates (on site)</u>

Latitude = 32° 50' 16.89645" N	Northing = 183,662.115 ift	*<u>Estimated accuracy</u>*
Longitude = 109° 42' 47.93756" W	Easting = 226,629.942 ift	Horizontal = ±0.047 ift
Ellipsoidal height = 2815.734 ift	Elevation = 2904.040 ift	Vertical = ±0.068 ift

GLOSSARY

Below is a list of the abbreviations and terms used in this workbook. In the interest of brevity, the definitions are highly general and simplified. Please note also that this list gives only a portion of the terms and abbreviations frequently encountered in GPS positioning and geodesy. Terms in *italics* within the definitions are also defined in this list. Cited references are listed at the end of the workbook.

Autonomous position. A *GPS* position obtained with a single receiver using only the ranging capability of the *GPS* code (i.e., with no *differential correction*).

Cartesian coordinates. Coordinates based on a system of two or three mutually perpendicular axes. *Map projection* and *ECEF* coordinates are examples two- and three-dimensional Cartesian coordinates, respectively.

Confidence interval or level. A computed probability that the "true" value will fall within a specified region (e.g., 95% confidence level). Applies only to randomly distributed errors.

CORS (Continuously Operating Reference Stations). A nation-wide system of permanently mounted *GPS* antennas and receivers that collect *GPS* data continuously. The CORS network is extremely accurate and constitutes the primary survey control for the US. CORS data can be used to correct *GPS* survey and mapping results, and the data are freely available over the Internet.

Datum transformation. Mathematical method for converting one *geodetic* or *vertical datum* to another (there are several types, and they vary widely in accuracy).

Differential correction. A method for removing much of the error in an autonomous *GPS* position. Typically requires at least two simultaneously operating *GPS* receivers, with one of the two at a location of known geodetic coordinates.

ECEF (Earth-Centered, Earth-Fixed). Refers to a global three-dimensional (X, Y, Z) *Cartesian coordinate* system with its origin at the Earth's center of mass, and "fixed" so that it rotates with the solid Earth. The Z-axis corresponds to the Earth's conventional spin axis, and the X- and Y-axes lie in the equatorial plane. Widely used for geodetic and *GPS* computations.

Ellipsoid. A simple mathematical model of the Earth corresponding to mean sea level (the *geoid*) and used as part of a *geodetic datum* definition. Constructed by rotating an ellipse about its semi-minor axis. Also referred to as a "spheroid".

Ellipsoid height. Straight-line height above and perpendicular to the *ellipsoid*. This is the type of height determined by *GPS*, and it does not equal elevation. Can be converted to orthometric heights ("elevations") using a *geoid* model.

Ellipsoid normal. A line perpendicular to the reference *ellipsoid* along which *ellipsoid heights* are measured.

FBN (Federal Base Network). Nationwide network of *GPS* control stations observed and adjusted by the *NGS*. A nation-wide readjustment of the FBN is scheduled for 2007.

FGDC (Federal Geographic Data Committee). Develops and promulgates information on spatial data formats, accuracy, specifications, and standards. Widely referenced by other organizations. Includes the Federal Geodetic Control Subcommittee (FGCS) and the *NSSDA*.

Geodetic datum. Reference frame for computing geodetic coordinates (latitude, longitude, and ellipsoid height) of a point. A datum always refers to a particular *ellipsoid* and a specific adjustment (e.g. the 1992 adjustment of *NAD 83* for the Arizona *HARN*).

Geographic "projection". This is not a true *map projection* in the sense that it does not transform geodetic coordinates (latitude and longitude) into linear units. However, it is a projection in the sense that it represents geodetic coordinates on a regular flat grid, such that the difference in angular units (e.g., decimal degrees) is equal in all directions. Because of meridian convergence, this results in an extremely distorted coordinate system, especially at high latitudes, and the distortion varies greatly with direction.

Geoid. Surface of constant gravitational equipotential (a level surface) that best corresponds to global mean sea level. Often used as a reference surface for *vertical datums*.

GPS (Global Positioning System). A constellation of satellites used for navigation, mapping, surveying, and timing. Microwave signals transmitted by the satellites are observed by GPS receivers to determine a three-dimensional position. Accuracy varies greatly depending on the type of receiver and methods used.

Grid distance. The horizontal distance between two points on a flat plane. This is the type of distance obtained from *map projections*.

Ground distance. The horizontal distance between two points as measured on the curved Earth surface.

GRS-80 (Geodetic Reference System of 1980). The reference *ellipsoid* currently used for many *geodetic datums* throughout the world, including *NAD 83* and *ITRF*.

HARN (High Accuracy Reference Network). Network of *GPS* stations adjusted by the *NGS* on a state-by-state basis. The Arizona HARN was adjusted in 1992. In some states it is referred to as a High Precision *GPS* (or Geodetic) Network (HPGN).

International Foot. Linear unit adopted by the US in 1959, and defined such that one foot equals exactly 0.3048 meter. Shorter than the *US Survey Foot* by 2 *parts per million* (ppm).

ITRF (International Terrestrial Reference Frame). Global geodetic reference system that takes into account plate tectonics (continental drift) and is used mainly in scientific studies. A new ITRF "epoch" is computed periodically and is referenced to a specific time (e.g., ITRF 2000 1997.0). Each epoch is a realization of the International Terrestrial Reference System (ITRS). See Soler (2007), and Soler and Snay (2004) for information on its relationship to *NAD 83* and *WGS 84*.

Local geodetic horizon. A "northing", "easting", and "up" planar coordinate system defined at a point such that the northing-easting plane is perpendicular to the *ellipsoid normal*, north corresponds to true geodetic north, and "up" is in the direction of the *ellipsoid normal* at that point.

Map projection. A functional (one-to-one) mathematical relationship between geodetic coordinates (latitude, longitude) on the curved *ellipsoid* surface, and grid coordinates (northings, eastings) on a planar (flat) map surface. All projections are distorted, in that the relationship between projected coordinates differs from that between their respective geodetic coordinates. See Snyder (1987) for details.

NAD 27 (North American Datum of 1927). *Geodetic datum* of the US prior to *NAD 83*, and superseded by *NAD 83* in 1986. This is the datum of *SPCS* 27 and *UTM* 27.

NAD 83 (North American Datum of 1983). Current official *geodetic datum* of the US. Replaced *NAD 27* in 1986, which is the year of the initial NAD 83 adjustment. This is the datum of *SPCS* 83 and *UTM* 83. See Schwarz (1986) for details.

NADCON. *Datum transformation* computer program developed by the NGS for transforming coordinates between *NAD 27* and *NAD 83*, and also between the *NAD 83* 1986 adjustment and the various *HARN* adjustments. See Dewhurst (1990) for details.

NAVD 88 (North American Vertical Datum of 1988). Current official vertical datum of the US. Replaced *NGVD 29* in 1991. See Zilkoski et al. (1992) for details.

NDGPS (National Differential GPS). A nation-wide system of "beacons" (permanently mounted *GPS* receivers and radio transmission equipment) that transmits real-time *differential corrections* which can be used by *GPS* receivers equipped with the appropriate radio receivers. Operated and maintained by the US Coast Guard. See US Coast Guard (2004) for details.

NGS (National Geodetic Survey). Federal agency within the Department of Commerce responsible for defining, maintaining, and promulgating the *NSRS* within the US and its territories.

NGVD 29 (National Geodetic Vertical Datum of 1929). Previous *vertical datum* of the US, superseded by *NAVD 88* in 1991. Not referenced to the *geoid* or mean sea level, and not as compatible with *GPS*-derived elevations as *NAVD 88*. Called "Mean Sea Level" (MSL) datum prior to 1976.

NSRS (National Spatial Reference System). The framework for latitude, longitude, height, scale, gravity, orientation and shoreline throughout the US. Consists of geodetic control point coordinates and sets of models describing relevant geophysical characteristics of the Earth, such as the *geoid* and surface gravity. Defined, maintained, and promulgated by the *NGS* (see Doyle, 1994, for details).

NSSDA (National Standard for Spatial Data Accuracy). *FGDC* methodology for determining the positional accuracy of spatial data (see Federal Geographic Data Committee, 1998).

OPUS (Online Positioning User Service). A free *NGS* service that computes *NSRS* and *ITRF* coordinates with respect to the *CORS* using raw *GPS* data submitted via the Internet.

Orthometric correction. A correction applied to leveled height differences which reduces systematic errors due to variation in gravitational potential. See Dennis (2004) for details.

Parts per million (ppm). A method for conveniently expressing small numbers, accomplished by multiplying the number by 1 million (e.g., 0.00001 = 10 ppm). Exactly analogous to percent, which is "parts per hundred".

SPCS (State Plane Coordinate System). A system of standardized *map projections* covering each state with one or more zones such that a specific distortion criterion is met (usually 1:10,000). Projection parameters (including units of length) are independently established by the legislature of each state. Can be referenced to either the *NAD 83* or *NAD 27* datums (SPCS 83 and SPCS 27, respectively). See Stem (1989) for details.

Triangulation. A method for determining positions from angles measured between points (requires at least one distance to provide scale).

Trilateration. A method for determining positions from measured distances only.

Trivial vector. A *GPS* vector (computed line connecting two *GPS* stations) that is not statistically independent from other *GPS* vectors observed at the same time.

US Survey Foot. Linear unit of the US prior to 1959, and defined such that one foot equals exactly 1200 / 3937 meter. Longer than the *International Foot* by 2 *parts per million* (ppm).

UTM (Universal Transverse Mercator). A grid coordinate system based on the Transverse Mercator *map projection* which divides the Earth (minus the polar regions) into 120 zones in order to keep map scale error within 1:2500. Can be referenced to either the *NAD 83* or *NAD 27* datums (UTM 83 and UTM 27, respectively). See Hager et al. (1989) for details.

Vertical datum. Reference system for determining "elevations", typically through optical leveling. Modern vertical datums typically use the *geoid* as a reference surface and allow elevation determination using *GPS* when combined with a *geoid* model.

WAAS (Wide Area Augmentation System). A system of geosynchronous satellites and ground *GPS* reference stations developed and managed by the Federal Aviation Administration and used to provide free real-time *differential corrections*. See Federal Aviation Administration (2003) for details.

WGS 84 (World Geodetic System of 1984). Reference *ellipsoid* and *geodetic datum* of *GPS*, defined and maintained by the US Department of Defense. Current realizations of WGS 84 are considered identical to *ITRF* 2000 at the 2 cm level. See National Imagery and Mapping Agency (1997) for details, and Merrigan et al. (2002) for information on the most recent realization.

SELECTED GPS AND GEODESY REFERENCES

Primary resource: The National Geodetic Survey (**http://www.ngs.noaa.gov/**)

Some NGS web pages of particular interest

Control station datasheets: **http://www.ngs.noaa.gov/cgi-bin/datasheet.prl**

The Geodetic Tool Kit: **http://www.ngs.noaa.gov/TOOLS/**

Online Positioning User Service (OPUS): **http://www.ngs.noaa.gov/OPUS/**

Continuously Operating Reference Stations (CORS): **http://www.ngs.noaa.gov/CORS/**

The Geoid Page: **http://www.ngs.noaa.gov/GEOID/**

NGS State Geodetic Advisors: **http://www.ngs.noaa.gov/ADVISORS/AdvisorsIndex.shtml**

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