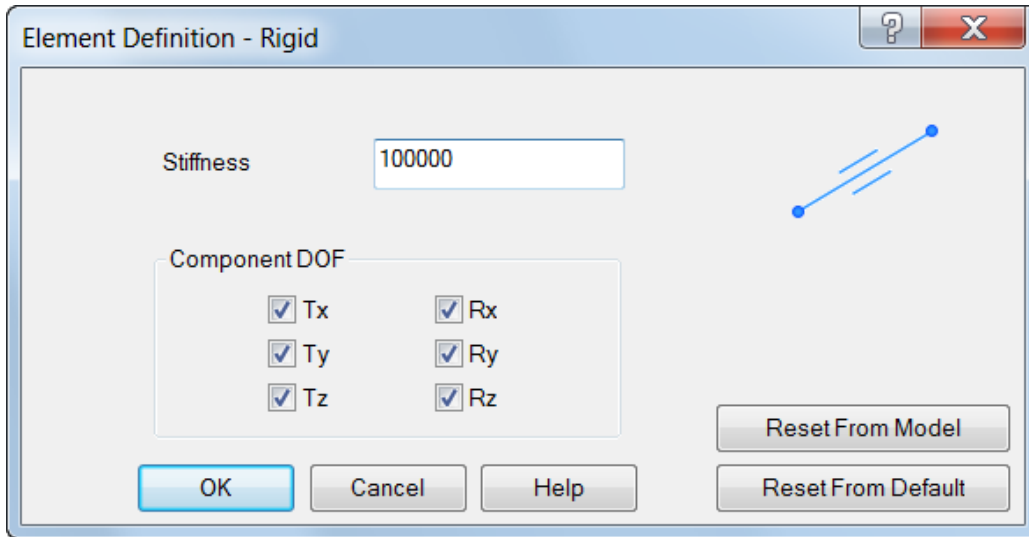


## Simulation Mechanical Rigid Element



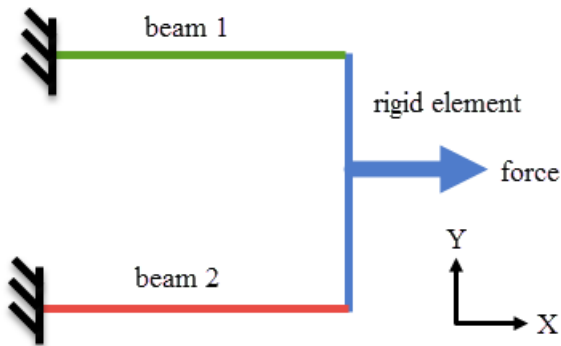
<b>Geometry</b>	One node (“master”) is connected to multiple nodes (“slaves”).
<b>Stiffness</b>	Defined by the user.
<b>Displacements of slave nodes</b>	<ul style="list-style-type: none"> <li>• If degree of freedom (“component DOF”) is checked, the displacement (or rotation) of the slave node in that direction is based on the motion of the master node. (This is explained below through the conceptual description.)</li> <li>• If degree of freedom is unchecked, the displacement of the slave node in that direction is not constrained by the motion of the master node.</li> </ul>
<b>Forces at slave nodes</b>	<ul style="list-style-type: none"> <li>• If degree of freedom is checked, the force (or moment) at the slave node in that direction is based on the solution of the model.</li> <li>• If degree of freedom is unchecked, the force or moment at the slave node in that direction is zero.</li> </ul>
<b>Loads and constraints on master node</b>	Any load or constraint that is in the component DOF has an effect. Other loads and constraints are ignored.
<b>Element in equilibrium?</b>	Yes, but only in selected DOF.

### Conceptual Description

Imagine the rigid element as a separate rigid body that is connected to the model by a series of springs. These springs act only in the direction of the component DOF.

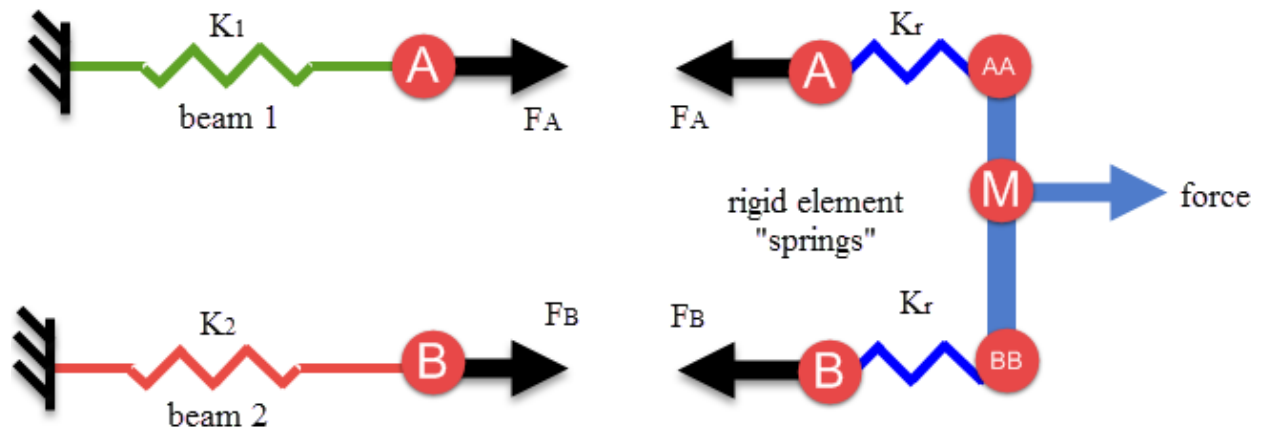
Think of two sets of slave nodes: one set “on the rigid element” and one set “on the model”. The separate rigid body moves only in the direction of the selected DOF. The forces transmitted to the model are the result of the displacement of the slave nodes on the rigid element relative to the slave node on the model and the stiffness of the rigid element.

Example 1 – rigid element set to Tx (translation)

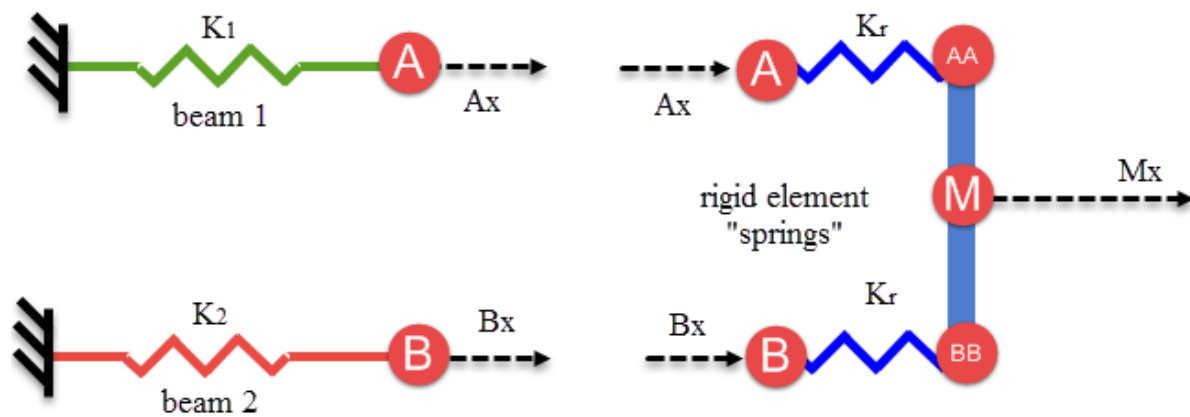


- Axial stiffness of beam 1,  $K_1 = 10$
- Axial stiffness of beam 2,  $K_2 = 8$
- Force = 100
- Stiffness of rigid element,  $K_r = 10000$  (unless noted otherwise)
- Component DOF: Tx

Free body diagram showing forces:



Free body diagram showing displacements:



Note: subscript x is translation

Equations of equilibrium:

For the beams

$$FA=K1 \cdot Ax$$

$$FB=K2 \cdot Bx$$

For the rigid elements

$$AAx=Mx$$

$$BBx=Mx$$

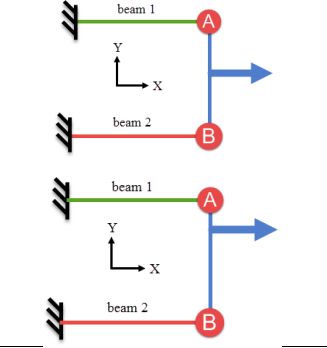
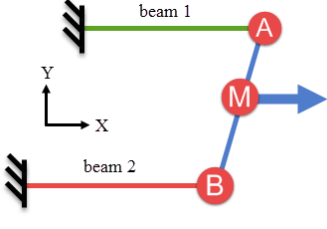
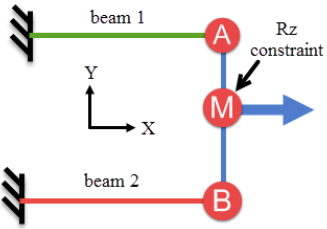
$$FA=Kr \cdot (AAx-Ax)$$

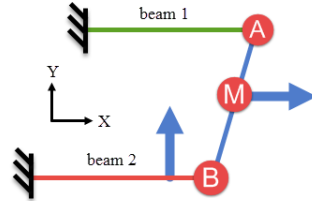
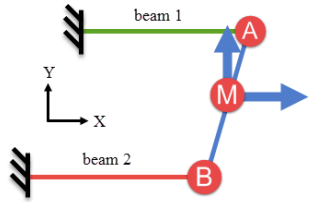
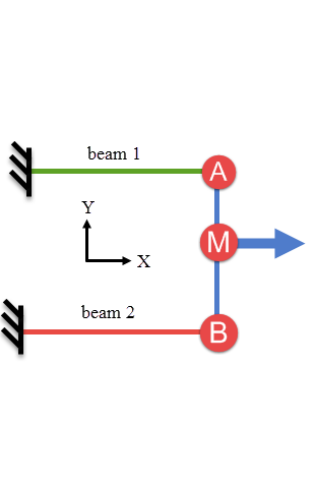
$$FB=Kr \cdot (BBx-Bx)$$

Rigid Element equilibrium

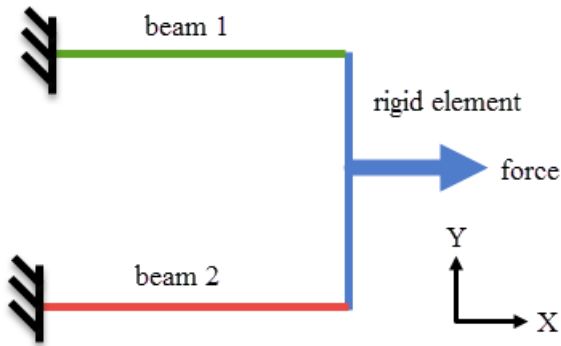
$$FA+FB=Force \text{ (X forces)}$$

- 7 equations, 7 unknowns.
- Note that the rigid link only translates in the X direction (that is, the selected “component DOF”). It does not rotate. Thus, the slave nodes (AA and BB) on the rigid element displace the same distance as the master node (M).
- Note that the rigid link is only in X force equilibrium (in the selected “component DOF” directions). It is not in equilibrium about the Z rotation (sum of moments is not zero).
- From the above equations, it can be shown that the slave nodes move the same distance as the master node if the rigid element has a high stiffness. If the rigid element is not stiff, the “rigid” springs elongate, and the displacement of the slave nodes is less than the master node. Specifically,  $FA=K1 \cdot Ax=Kr \cdot (Mx-Ax)$ , so  $Ax=Mx \cdot Kr / (K1+Kr)$ .

Example 1	Simulation Mechanical Results (displacements)
<p>Q: Does the position of the load affect the solution?</p> <p>A: No. The rigid element is only in force equilibrium in the X direction (the selected “component DOF”), so the position of the load has no effect.</p>	 <p style="text-align: right;"> <math>Ax = 5.56</math>  <math>Ay = 0</math>  <math>Bx = 5.56</math>  <math>By = 0</math> </p>
<p>Q: Does the orientation (vertical versus sloped) of the rigid element affect the solution?</p> <p>A: No. The springs of the rigid element are treated as if they are horizontal.</p>	 <p style="text-align: right;"> <math>Ax = 5.56</math>  <math>Ay = 0</math>  <math>Bx = 5.56</math>  <math>By = 0</math> </p>
<p>Q: Does adding a rotational constraint on the master node affect the solution?</p> <p>A: No. The elements are only transmitting axial forces.</p>	 <p style="text-align: right;"> <math>Ax = 5.56</math>  <math>Ay = 0</math>  <math>Bx = 5.56</math>  <math>By = 0</math> </p>

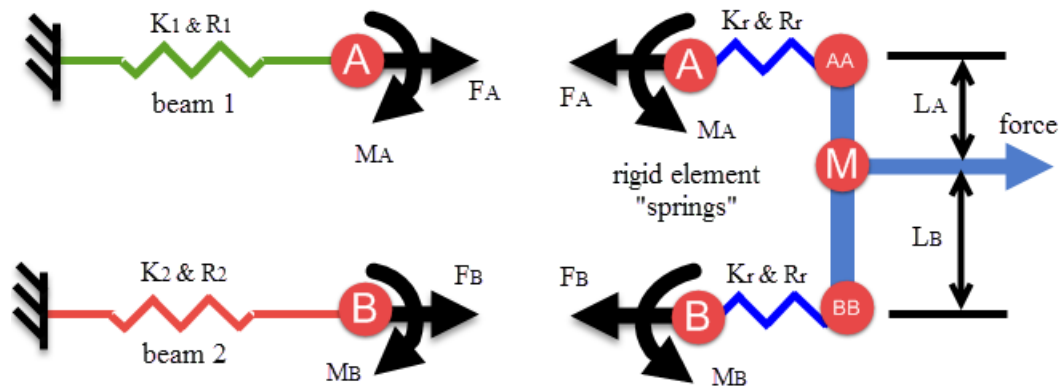
Example 1	Simulation Mechanical Results (displacements)	
<p>Q: Does a vertical load on one of the beams affect the other beam?</p> <p>A: No. The rigid element is only transmitting forces in the horizontal direction.</p>		<p>Ax = 5.56 Ay = 0 Bx = 5.56 By = some value</p>
<p>Q: Does a vertical load on the rigid element affect the solution?</p> <p>A: No. The rigid elements are only transmitting horizontal forces.</p>		<p>Ax = 5.56 Ay = 0 Bx = 5.56 By = 0</p>
<p>Q: How does the stiffness of the rigid element affect the solution?</p> <p>A: The stiffness of the "rigid" element affects the force transmitted to the model and therefore the displacements.</p>		<p><u>Kr = 1</u> Ax = 5.06 Ay = 0 Mx = 55.62 My = 0 Bx = 6.18 By = 0 <u>Kr = 100</u> Ax = 5.51 Ay = 0 Mx = 6.06 My = 0 Bx = 5.61 By = 0</p>

Example 2 – rigid element set to Tx Rz (translation and rotation)



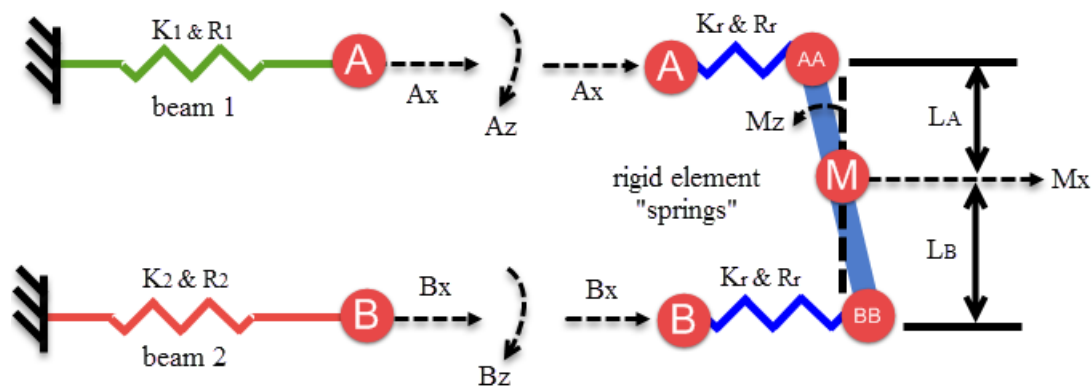
- Axial stiffness of beam 1, K1 = 10
- Rotation stiffness of beam 1, R1 = 0.833
- Axial stiffness of beam 2, K2 = 8
- Rotation stiffness of beam 2, R2 = 0.667
- Force = 10
- Stiffness of rigid element, Kr=Rr= 10000 (unless noted otherwise)
- Component DOF: Tx & Rz

Free body diagram showing forces and moments:



Note: K is axial stiffness, R is rotational stiffness

Free body diagram showing displacements and rotations:



Note: subscript x is translation, subscript z is rotation

Equations of equilibrium:

For the beams

$$F_A = K_1 \cdot A_x$$

$$F_B = K_2 \cdot B_x$$

$$M_A = R_1 \cdot A_z$$

$$M_B = R_2 \cdot B_z$$

For the rigid elements

$$A A_x = M_x - L_A \cdot 0.5 \cdot (M_z + A_z)$$

$$B B_x = M_x + L_B \cdot 0.5 \cdot (M_z + B_z)$$

$$A_x = A A_x - F_A / K_r$$

$$B_x = B B_x - F_B / K_r$$

$$A A_z = M_z$$

$$B B_z = M_z$$

$$M_A = R_r \cdot (A A_z - A_z)$$

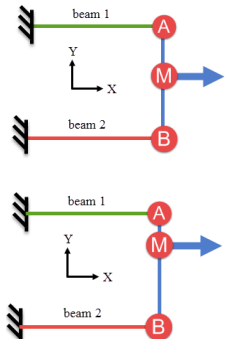
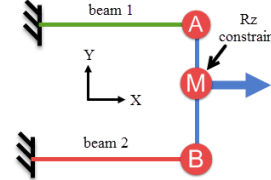
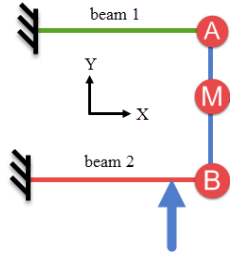
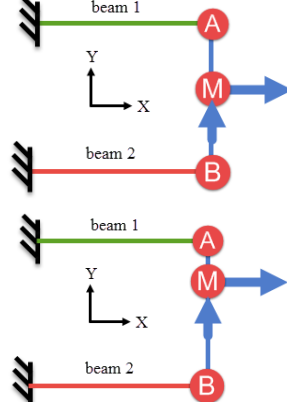
$$M_B = R_r \cdot (B B_z - B_z)$$

Rigid Element equilibrium

$$F_A + F_B = \text{Force (X forces)}$$

$$F_A \cdot L_A + M_A + M_B = F_B \cdot L_B \text{ (moment about M)}$$

- 14 equations, 14 unknowns.
- Note that the rigid link translates in the X direction and rotates about the Z direction (that is, the selected "component DOF").
- Note that the rigid link is in X force equilibrium and Z moment equilibrium (in the selected "component DOF" directions).

Example 2	Simulation Mechanical Results (displacements and rotations)																																	
<p>Q: Does the position of the load affect the solution?</p> <p>A: Yes. Because the rigid element can rotate and is in moment equilibrium, the position of the load affects the solution.</p>		<p>Force at center</p> <table border="1" data-bbox="1015 262 1380 409"> <thead> <tr> <th></th> <th>x</th> <th>y</th> <th>z</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>0.514</td> <td>0.050</td> <td>5.681°</td> </tr> <tr> <td>M</td> <td>0.563</td> <td>0</td> <td>5.350°</td> </tr> <tr> <td>B</td> <td>0.607</td> <td>0.044</td> <td>5.000°</td> </tr> </tbody> </table> <p>Force 20% from A to B</p> <table border="1" data-bbox="1015 451 1380 598"> <thead> <tr> <th></th> <th>x</th> <th>y</th> <th>z</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>0.740</td> <td>-0.193</td> <td>-22.115°</td> </tr> <tr> <td>M</td> <td>0.660</td> <td>0</td> <td>-23.940°</td> </tr> <tr> <td>B</td> <td>0.325</td> <td>-0.210</td> <td>-24.022°</td> </tr> </tbody> </table>		x	y	z	A	0.514	0.050	5.681°	M	0.563	0	5.350°	B	0.607	0.044	5.000°		x	y	z	A	0.740	-0.193	-22.115°	M	0.660	0	-23.940°	B	0.325	-0.210	-24.022°
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<p>Q: Does a vertical load on one of the beams affect the other beam?</p> <p>A: Yes. The force on the beam changes the rotation of the beam, and the rigid element is transmitting a moment load. So the vertical load is indirectly affecting the other beams.</p>		<p>Results depend on magnitude and location of vertical force.</p>																																
<p>Q: Does a vertical load on the rigid element affect the solution?</p> <p>A: No. The vertical force on the rigid element does not create any load that is transmitted by the rigid element.</p>		<p>Force at center</p> <table border="1" data-bbox="1015 1134 1380 1281"> <thead> <tr> <th></th> <th>x</th> <th>y</th> <th>z</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>0.514</td> <td>0.050</td> <td>5.681°</td> </tr> <tr> <td>M</td> <td>0.563</td> <td>0</td> <td>5.350°</td> </tr> <tr> <td>B</td> <td>0.607</td> <td>0.044</td> <td>5.000°</td> </tr> </tbody> </table> <p>Force 20% from A to B</p> <table border="1" data-bbox="1015 1333 1380 1491"> <thead> <tr> <th></th> <th>x</th> <th>y</th> <th>z</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>0.740</td> <td>-0.193</td> <td>-22.115°</td> </tr> <tr> <td>M</td> <td>0.660</td> <td>0</td> <td>-23.940°</td> </tr> <tr> <td>B</td> <td>0.325</td> <td>-0.210</td> <td>-24.022°</td> </tr> </tbody> </table>		x	y	z	A	0.514	0.050	5.681°	M	0.563	0	5.350°	B	0.607	0.044	5.000°		x	y	z	A	0.740	-0.193	-22.115°	M	0.660	0	-23.940°	B	0.325	-0.210	-24.022°
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Revisions

Rev	Date	Change
0	2016 Sep 28	Initial release. (JWH)