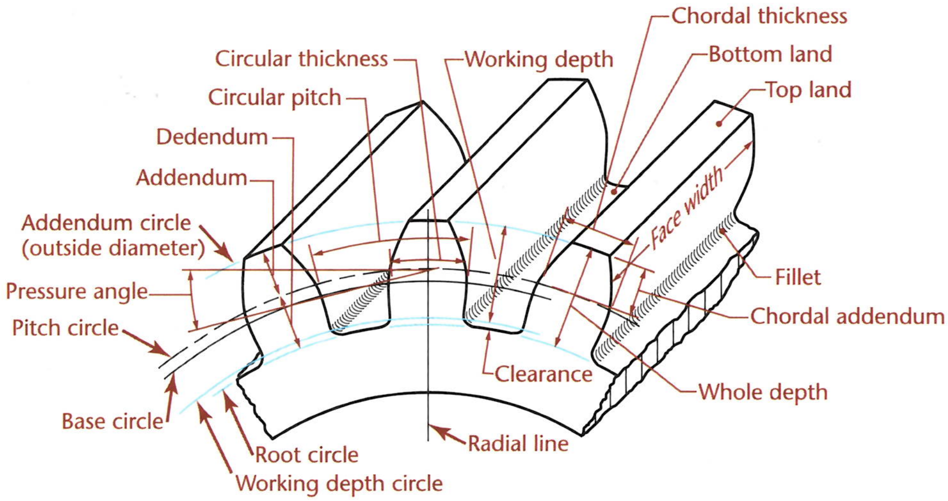
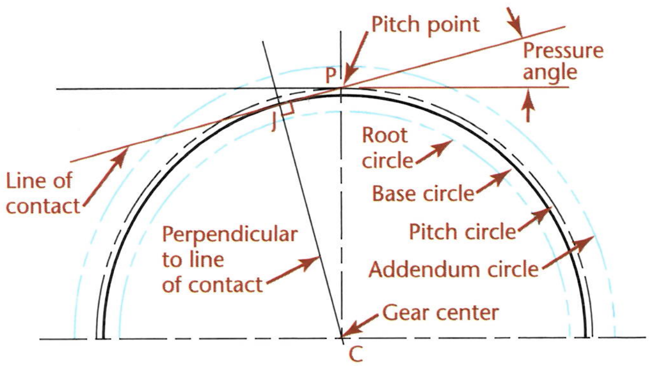
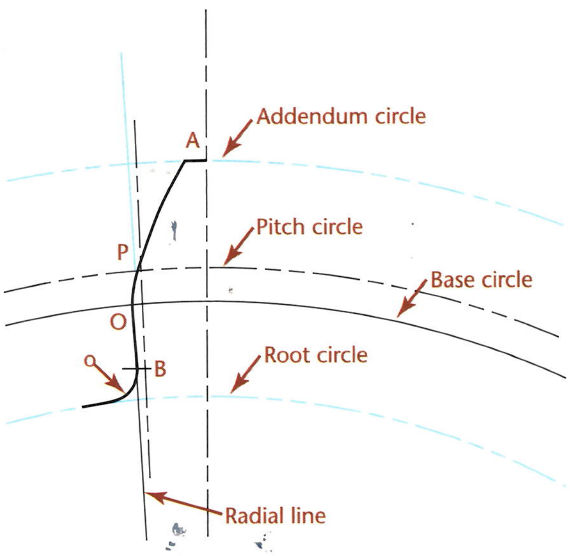


Term	Symbol	Definition	Formula for 20°/25° Pressure Angles	Formula for 14.5° Pressure Angle
Addendum	a	Radial distance from pitch circle to top of tooth	$a = 1/P$	$a = 1/P$
Base circle	D_B	Circle from which involute profile is generated	$D \cos(20° \text{ or } 25°)$	$D \cos(14.5°)$
Chordal addendum	a_c	Radial distance from the top of a tooth to the chord of the pitch circle	$a_c = a + \frac{1}{2} D [1 - \cos(90°/N)]$	$a_c = a + \frac{1}{2} D [1 - \cos(90°/N)]$
Chordal thickness	t_c	Thickness of a tooth measured along a chord of the pitch circle	$t_c = D \sin(90°/N)$	$t_c = D \sin(90°/N)$
Circular pitch	p	Distance measured along pitch circle from a point on one tooth to corresponding point on the adjacent tooth; includes one tooth and one space	$p = \pi D/N$ $p = \pi/P$	$p = \pi D/N$ $p = \pi/P$
Circular thickness	t	Thickness of a tooth measured along the pitch circle: equal to half the circular pitch	$t = p/2 = \pi/2P$ $t = \pi/2P$	$t = p/2 = \pi/2P$ $t = \pi/2P$
Clearance	c	Distance between top of a tooth and bottom of mating space, equal to the dedendum minus the addendum	$c = b - a$ $c = 0.25/P$	$c = b - a$ $c = 0.157/P$
Dedendum	b	Radial distance from pitch circle to bottom of tooth space	$b = 1.250/P$	$b = 1.157/P$
Diametral pitch	P	Ratio equal to the number of teeth on the gear per inch of pitch diameter	$P = N/D$	$P = N/D$
Number of teeth	N_G or N_P	Number of teeth on the gear or pinion	$N = P \times D$	$N = P \times D$
Outside diameter	D_o	Diameter of addendum circle, equal to pitch diameter plus twice the addendum	$D_o = D + 2a$ $D_o = (N + 2)/P$	$D_o = D + 2a$ $D_o = (N + 2)/P$
Pitch circle		An imaginary circle with the circumference of the friction gear from which the spur gear is derived		
Pitch diameter	D_G or D_P	Diameter of pitch circle of gear or pinion	$D = N/P$	$D = N/P$
Pressure angle	ϕ	Angle that determines direction of pressure between contacting teeth and designates shape of involute teeth—e.g. 14.5°, 20° or 25° involute; also determines the size of base circle		
Root diameter	D_R	Diameter of the root circle; equal to pitch diameter minus twice the dedendum	$D_R = D - 2b = (N - 2.5)/P$	$D_R = D - 2b = (N - 2.314)/P$
Whole depth	h_t	Total height of the tooth; equal to the addendum plus the dedendum	$h_t = a + b = 2.250/P$	$h_t = a + b = 2.157/P$
Working depth	h_k	Distance a tooth projects into mating space; equal to twice the addendum	$h_k = 2a = 2/P$	$h_k = 2a = 2/P$

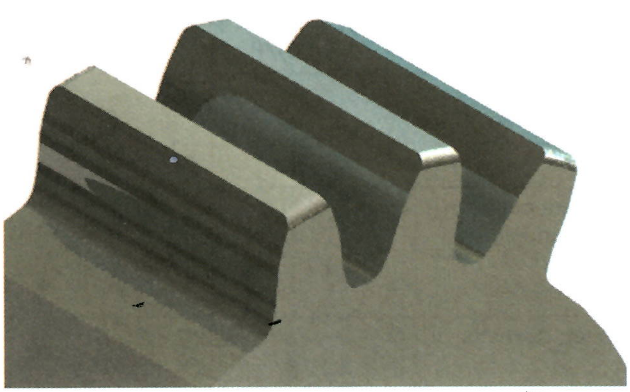
16.2 Spur Gear Terminology



16.3 Construction of a Base Circle



16.4 The Involute Profile



16.5 Shaded Model of Involute Shaped Spur Gear Teeth. *It is not typical to model gear teeth in detail, as doing so creates an unnecessarily large and complex model.*

To make gears operate smoothly with a minimum of noise and vibration, the curved surface of the tooth profile uses a definite geometric form. The most common form in use today is the ***involute*** profile shown in Figure 16.4. (The word *involute* means "rolled inward.")

16.1 CONSTRUCTING A BASE CIRCLE

The involute tooth form depends on the pressure angle, which was ordinarily 14.5° and is now typically 20° or 25°. This pressure angle determines the size of the ***base circle;*** from this the involute curve is generated.

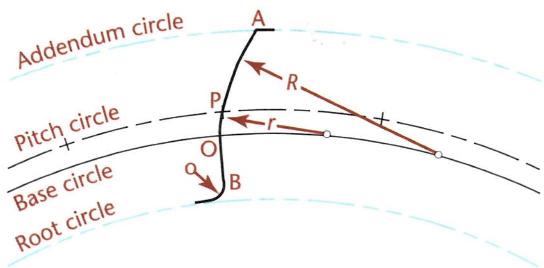
To calculate the base circle for the spur gear as shown in Figure 16.3, follow these steps. At any point on the pitch circle, such as point *P* (the pitch point) draw a line tangent to the pitch circle; draw a second line through *P* at the required pressure angle (frequently approximated at 15° on the drawing). This line is called the *line of contact*. Next, draw a line perpendicular to the line of contact from the center, *C*. Then, draw the base circle with radius *CJ* tangent to the line of contact at *J*.

16.2 THE INVOLUTE TOOTH SHAPE

If the exact shape of the tooth is desired, the portion of the profile from the base circle to the ***addendum circle*** (outside diameter) can be drawn as the involute of the base circle. In Figure 16.4, the tooth profile from *A* to *O* is an involute of the base circle. The part of the profile below the base circle, line *OB*, is drawn as a radial line (a straight line drawn from the gear center) that terminates in the fillet at the ***root circle.*** The fillet should be equal in radius to one and a half times the clearance from the tip of the tooth to the bottom of the mating space.

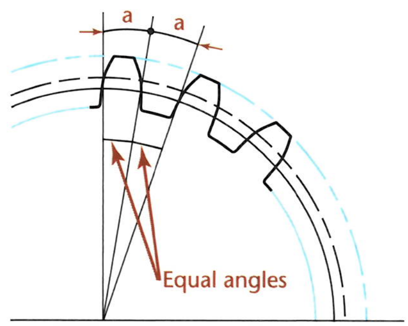
16.3 APPROXIMATE INVOLUTE USING CIRCULAR ARCS

Involute curves can be closely approximated with two circular arcs, as shown in Figure 16.6. This method, originally devised by G. B. Grant, uses a table of arc radii, an involute odontograph, for gears with various numbers of teeth. To use this method, draw the base circle as described above, and set off the spacing of the teeth along the pitch circle. Then, draw the face of the tooth from *P* to *A* with the face radius *R*, and draw the portion of the flank from *P* to *O* with the flank radius *r*. Draw both arcs from centers located on the base circle. The table in Figure 16.6 gives the correct face and flank radii for gears of one ***diametral pitch.*** For other pitches, divide the values in the table by the diametral pitch. For gears with more than 90 teeth, use a single radius (let $R = r$) computed from the appropriate formula given in Figure 16.6, then divide by diametral pitch. Below the base circle, complete the flank of the tooth with a radial line *OB* and a fillet. (A typical fillet radius is 1.5 times the clearance.)



No. of Teeth (*N*)	14.5°		20°	
	R (in.)	*r* (in.)	*R* (in.)	*r* (in.)
12	2.87	0.79	3.21	1.31
13	3.02	0.88	3.40	1.45
14	3.17	0.97	3.58	1.60
15	3.31	1.06	3.76	1.75
16	3.46	1.16	3.94	1.90
17	3.60	1.26	4.12	2.05
18	3.74	1.36	4.30	2.20
19	3.88	1.46	4.48	2.35
20	4.02	1.56	4.66	2.51
21	4.16	1.66	4.84	2.66
22	4.29	1.77	5.02	2.82
23	4.43	1.87	5.20	2.98
24	4.57	1.98	5.37	3.14
25	4.70	2.08	5.55	3.29
26	4.84	2.19	5.73	3.45
27	4.97	2.30	5.90	3.61
28	5.11	2.41	6.08	3.77
29	5.24	2.52	6.25	3.93
30	5.37	2.63	6.43	4.10
31	5.51	2.74	6.60	4.26
32	5.64	2.85	6.78	4.42
33	5.77	2.96	6.95	4.58
34	5.90	3.07	7.13	4.74
35	6.03	3.18	7.30	4.91
36	6.17	3.29	7.47	5.07
37–39	6.36	3.46	7.82	5.32
40–44	6.82	3.86	8.52	5.90
45–50	7.50	4.46	9.48	6.76
51–60	8.40	5.28	10.84	7.92
61–72	9.76	6.54	12.76	9.68
73–90	11.42	8.14	15.32	11.96
91–120	0.118*N*		0.156*N*	
121–180	0.122*N*		0.165*N*	
Over 180	0.125*N*		0.171*N*	

16.6 Wellman's Involute Odontograph for Drawing Gear Teeth Using Circular Arcs



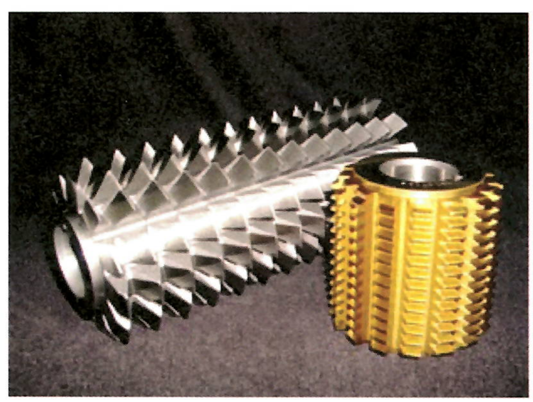
16.7 Spacing Gear Teeth

16.4 SPACING GEAR TEETH

Suppose that number of teeth (N) = 20 and diametral pitch (P) = 4 for a 14.5° involute tooth. The values from Figure 16.6 are $R = 4.02$ and $r = 1.56$. These must be divided by P; yielding $R = 4.02/4 = 1.005''$ and $r = 1.56/4 = 0.39''$.

Space the teeth around the periphery by laying out equal angles (Figure 16.7). The number of spaces should be $2N$, twice the number of teeth, to make the space between teeth equal to the tooth thickness at the pitch circle. In the example, because $N = 20$, $a = 360°/2N = 360°/40 = 9°$, the angle subtended by each tooth and each space.

> **TIP**
>
> The Divide command in the AutoCAD software is very handy for dividing a circle or other geometry into any number of equal divisions. You can also use it to insert a block at the same time. A polar array is another useful tool for creating gears.



Involute Gear Hob. *Spur gears with involute tooth shapes are usually manufactured using a hobbing machine. This machine cuts gear teeth by rotating the gear blank and a cutter like the one shown at a fixed speed ratio. The cross-sectional profile of the sides of the teeth on the cutter generates the involute tooth shape for the gear. Very small gears normally must be milled instead.* (Courtesy of Hobsource.)