Formulas used in ROBOT modules to calculate cross-sectional properties of the user-defined profiles (boxes, tubes, I-shapes)

General remarks:

A complete set of formulas is given in the following tables. The tables are divided into four columns:

the 1st column	-	quantity symbol,
the 2nd column	-	formula
the 3rd column	-	comments (if necessary) about known incorrectness and planned modifications. The “FUNCTION” mark means planned replacement of the current pattern by the similar (precise) function of the characteristic diameters.
the 4th column	-	comments about formula origin (literature, where the formula may be found), and its precision with comparison to exact value.

All the definitions may describe the “thin” and “thick” profiles (in extrem the solid profiles may be defined). Hence, the separate formulas were used for both types of profiles (thin/thick). All the formulas were evaluated for the prismatic, elastic bars.

From all the quantities given below, only the Ax area is taken into account during the structure calculations (for Timoshenko’s beam model, as well Ay, Az). Others are used to estimate the stress values after the intrernal forces are known.

Bibliography:

EG	-	Elementary geometrical formulas
EM	-	Elementary formulas of the theory of elasticity
EYR	-	Handbook of steel constructions (“Guide practique de charpente metallique” - R.Daussy - Eyrolles, 1987)
RCM	-	Steel code “Regles de calculs des constructions en acier”, Eyrolles, 1986
TIM	-	“Theory of Elasticity” - Timoshenko, Goodier - McGraw-Hill, 1951
WASH	-	“Variational Methods in Elasticity and Plasticity” - Washizu, Pergamon Press, 1975

Symbols and names:

Ax	-	cross-sectional area
Ay	-	reduced cross-sectional area to calculate “shear rigidity” (influence of the shearing force Fy on beam deflections)
Az	-	reduced cross-sectional area to calculate “shear rigidity” (influence of the shearing force Fz on beam deflections)
Wx	-	torsional modulus ($\tau max=Mx/Wx$)
Wy	-	reduced shear area ($\tau ymax=Fy/Wy$)
Wz	-	reduced shear area ($\tau zmax=Fz/Wz$)

Definitions:

TUBE (pipe)
: R - external radius,
r - internal radius

THICK profiles (r < 0.83R)			
Ax	π *(R^2-r^2)		EG
Ay Az	27/32*Ax		WASH - precise only for solid section. Unknown exact value for thick hollow sect.
Wx	π *(R^4-r^4)/2R		TIM
Wy Wz	3/4*Ax	FUNCTION	RCM, TIM - precise only for solid section. Exact value in the range 0.75 - 0.5
THIN profiles (r > 0.83R)			
Ax	π *(R^2-r^2)		EG
Ay Az	0.5*Ax		WASH
Wx	π *(R^4-r^4)/2R		TIM
Wy Wz	0.5*Ax		RCM, TIM - exact value for r->R (over 0.9R practically stable value of 0.5Ax)

BOX:
h - web height
ea - web thickness
es - flange thickness
b - flange width

THICK profiles (Ax > 1/3 b*(h+2es))			
Ax	(h+2es)*b - h*(b-2ea)		EG
Ay Az	5/6*Ax		WASH - exact value for solid section. Unknown precise formula
Wx	0.23*Ax* min(b,h)		EYR - Weber's formula for solid section for others - unknown formula,
Wy Wz	2/3*Ax	FUNCTION	RCM, TIM - exact value for solid section Precise value in the range 0.667 - 0.450
THIN profiles (Ax <= 1/3b*(h+2es))			
Ax	(h+2es)*b - h*(b-2ea)		EG
Ay	2b*es		Unknown origin
Az	2(h+2es)* ea		Unknown origin
Wx	2*(h+es)*(b-ea) * min(ea,es)		Bredt's formula (EYR)
Wy	5/6*Ay	FUNCTION	Exact value depends on proportion b/h (appx. 0.89 Ay)

Wz	5/6*Az	FUNCTION	Exact value depends on proportion b/h (appx. 0.89 Ay)

RECT: h - total height
ep - web and flange thickness
b - flange width

THICK profiles (Ax > 1/3 b*h)			
Ax	h*b - (h-2ep)*(b-2ep)		EG
Ay Az	5/6*Ax		WASH - exact value for solid section Unknown precise formula
Wy Wz	2/3*Ax	FUNCTION	RCM, TIM - exact value for solid section precise formula in the range. 0.667 - 0.450
THIN profiles (Ax <= 1/3b*h)			
Ax	h*b -h*(b-2ep)		EG
Ay	2b*ep		Unkown origin
Az	2ep*h		Unkown origin
Wx	2*(h-ep)*(b-ep) *ep		Bredt's formula (EYR)
Wy	5/6*Ay	FUNCTION	Exact value depends on proportion b/h (appx. 0.89 Ay)
Wz	5/6*Az	FUNCTION	Exact value depends on proportion b/h (appx. 0.89 Az)

I-SECTION: h - web height
ea - web thickness
es - flange thickness
b - flange width

THICK profiles (Ax > 1/3 b*(h+2es))			
Ax	h*ea + 2b*es		EG
Ay Az	5/6*Ax		WASH - exact value for solid section Unknown precise formula
Wx	Ix / max(ea,es)		TIM - appx. formula
Wy Wz	2/3*Ax	FUNCTION	RCM, TIM - exact value for solid section precise formula in the range 0.667 - 0.450
THIN profiles (Ax <= 1/3b*(h+2es))			
Ax	h*ea + 2b*es		EG
Ay	2b*es		Flange area

Az	h*ea		Web area
Wx	Ix / max(ea,es)		TIM - appx. formula, for recatngles which ratio b/h > 2.5 (or < 0.4) - precise
Wy	8Iz/(b^2-ea^2)		Manual calculations on the base of EM (as well RCM)
Wz	8ea*Iy/ ((h+es)*b*4es+ h^2*ea)		Manual calculations on the base of EM (as well RCM)