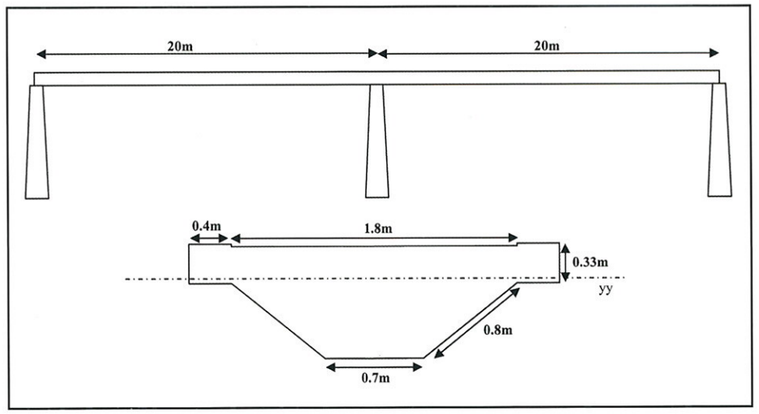
## 5.1 Example 1: A footbridge over a motorway

Example 1 is a concrete bridge over a motorway shown in Figure 5.1, and its main dimensions are shown in Figure 5.2.

**Figure 5.1**
**Worked example 1: footbridge over a motorway.**

**Figure 5.2**
**Approximate dimensions of bridge.**

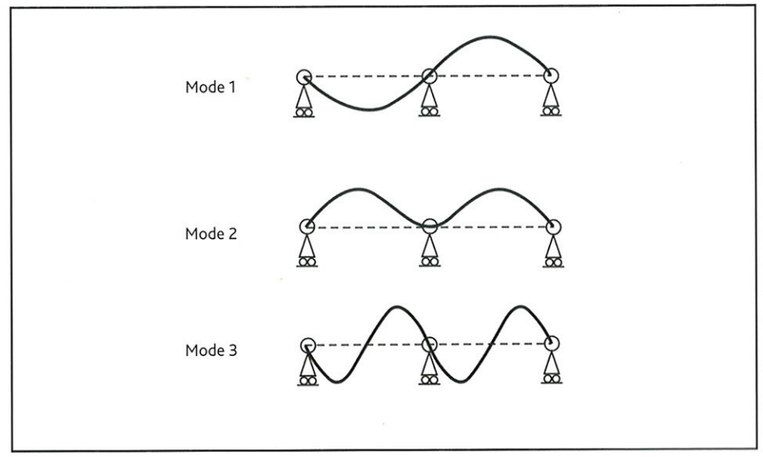





### 5.1.1 Calculation of modal properties

The cross-sectional area of the beam section is approximately 0.77m$^2$ and the second moment of area 0.056m$^4$. From these the modal properties can be calculated using either standard equations for the natural frequencies of beams or a simple finite element model. The shapes of the first three modes obtained from the finite element analysis are shown in Figure 5.3.

**Figure 5.3**
**Mode shapes of the first three modes of the footbridge.**




The natural frequencies and modal masses may be calculated by hand. Mode 1 is equivalent to the first simply supported mode of both spans, mode 2 is equivalent to the fixed-free mode of each span and mode 3 the second simply supported mode of both spans. Assuming un-cracked reinforced concrete with $E$ = 38GPa, and negligible non-structural mass, the mass per unit length is 1848kg/m and the natural frequencies obtained from the FE analysis and Equation A1 are identical. These are compared with measured values in Table 5.1. The modal mass of each mode is approximately 20 × 2 × 1848/2 = 36,960kg.

**Table 5.1**
**Calculated and measured natural frequencies**

| | Hand Calculation | Finite Element Solution | Measured Frequency |
|---|---|---|---|
| $f_1$ (Hz) | 4.22 | 4.22 | 4.65 |
| $f_2$ (Hz) | 6.59 | 6.59 | 6.56 |
| $f_3$ (Hz) | 16.90 | 16.88 | 14.66 |

### 5.1.2 Footfall induced vibration calculation

The bridge has two modes that might be excited to resonance by walking. Based on the measured natural frequencies, the first mode may be excited in either the second or third harmonic of walking at 2.32 or 1.55 footfalls per second respectively, and the second mode by the third or fourth harmonics of walking at 2.18 or 1.64 footfalls per second.