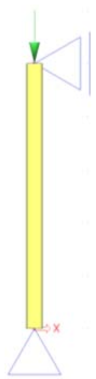
EN 1993-1-1 **article 5.3.2(11)**:
As an alternative the shape of the elastic buckling mode hcr of the structure may be applied as an unique global and local imperfection. The amplitude h_init of this imperfection may be determined from:

α - The imperfection factor for the relevant buckling curve.
χ - The reduction factor for the relevant buckling curve, depending on the relevant cross-section.
N_{Rk} - The characteristic resistance to normal force of the critical cross-section, i.e. Npl,Rk.
N_{cr} - Elastic critical buckling load.
M_{Rk} - The characteristic moment resistance of the critical cross-section, i.e. $M_{el,Rk}$ or $M_{pl,Rk}$ as relevant.
η_{cr} - Shape of the elastic critical buckling mode.
η''_{cr} - Maximal second derivative of the elastic critical buckling mode.

The column has a cross-section of type **IPE 300**, is fabricated from **S235** and has the following relevant
properties:



E = 210.000 N/mm²
fy = 235 N/mm²
γ_{M1} = 1.00
L = 5000 mm
A = 5380 mm²
Iy = 83560000mm^4
Wpl,y = 628400 mm^3

First a **Stability calculation** is done using a load of 1kN. This way, the elastic critical buckling load N_{cr} is obtained. In order to obtain precise results, the **Number of 1D elements** is set to **10**. In addition, the **Shear Force Deformation** is neglected so the result can be checked by a manual calculation.
The stability calculation gives the following result:

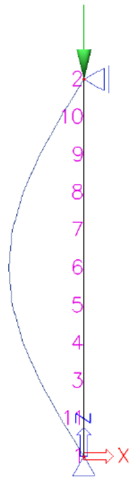
Critical load coefficients

Critical load coefficients	
N -	f []
Stability combination : S1	
1	6885,28

This can be verified with Euler's formula using the member length as the buckling length:

$$N_{cr} = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 210.000\ {}^{N}/_{mm^2} \times 83560000\ mm^4}{(5000\ mm)^2} = 6927.5\ kN$$

The following picture shows the mesh nodes of the column and the corresponding buckling shape:

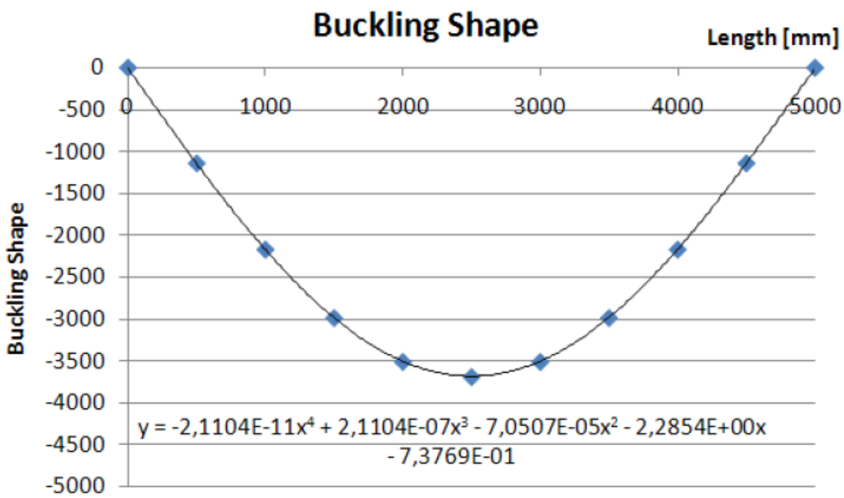


Displacement of nodes

Stability calculation, Extreme : No
Selection : All
Stability combinations : S1/1 - 6885,28
Values are mulitpied by 10000 for better numeric representation.

Node of mesh	Case	Ux [-]	Uz [-]	Fiy [-]
1	S1/1 - 6885,28	0,00	0,00	-2314,62
11	S1/1 - 6885,28	-1138,38	0,00	-2201,33
3	S1/1 - 6885,28	-2165,34	0,00	-1872,57
4	S1/1 - 6885,28	-2980,33	0,00	-1360,50
5	S1/1 - 6885,28	-3503,59	0,00	-715,26
6	S1/1 - 6885,28	-3683,89	0,00	0,00
7	S1/1 - 6885,28	-3503,59	0,00	715,26
8	S1/1 - 6885,28	-2980,33	0,00	1360,50
9	S1/1 - 6885,28	-2165,34	0,00	1872,57
10	S1/1 - 6885,28	-1138,38	0,00	2201,33
2	S1/1 - 6885,28	0,00	0,00	2314,62

Using for example an Excel worksheet, the buckling shape can be approximated by a 4th grade polynomial.



A polynomial has the advantage that the second derivative can easily be calculated.

$$\eta_{cr} = -2.1104 \times 10^{-11} \times x^4 + 2.1104 \times 10^{-7} \times x^4 - 7.0507 \times 10^{-5} \times x^2 + 2.2854 \times x - 7.3769 \times 10^{-1}$$

$$\eta''_{cr} = -2.5325 \times 10^{-10} \times x^2 + 1.2662 \times 10^{-6} \times x - 1.4101 \times 10^{-4}$$

Calculation of e_0

$$N_{Rk} = f_y \times A = 235\ {}^{N}/_{mm^2} \times 5380\ mm^2 = 1264300\ N$$

$$M_{Rk} = f_y \times W_{pl} = 235\ {}^{N}/_{mm^2} \times 628400\ mm^3 = 147674000\ Nmm\ (class\ 2)$$

$$\bar{\lambda} = \sqrt{\frac{N_{Rk}}{N_{cr}}} = \sqrt{\frac{1264300\ N}{6927510\ N}} = 0.43$$

$\alpha = 0.21$ for buckling curve a

$$\chi = \frac{1}{0.5\left[1+\alpha(\bar{\lambda}-0.2)+(\bar{\lambda})^2\right]+\sqrt{\left(0.5\left[1+\alpha(\bar{\lambda}-0.2)+(\bar{\lambda})^2\right]\right)^2-(\bar{\lambda})^2}} = 0.945$$

$$e_0 = \alpha(\bar{\lambda}-0.2)\frac{M_{Rk}}{N_{Rk}}\frac{1-\frac{\chi\times(\bar{\lambda})^2}{\gamma_{M1}}}{1-\chi\times(\bar{\lambda})^2} = 0.21\times(0.43-0.2)\frac{147674000\ Nmm}{1264300\ N} = 5.6416\ mm$$

Calculation of η_{init}

The mid section of the column is decisive => x = 2500 mm

η_{cr} at mid section = -3681,8

η''_{cr} at mid section = 1,4418 × 10^{-3} $1/mm^2$

$$\eta_{init} = e_0 \times \frac{N_{cr}}{E\times I\times\eta''_{cr}}\times\eta_{cr} = 5.6416\ mm \times \frac{6927510\ N}{210000\ {N}/{mm^2}\times 83560000\ mm^4\times 1.4418\times10^{-3}\times {1}/{mm^2}}\times 3681.8 = 5.6528\ mm$$

This value can now be inputted as amplitude of the buckling shape for imperfection.