FOR 2D FRAMES

The consistent mass matrix:

$$
m=\rho l\begin{bmatrix}
\frac{A}{3} & 0 & 0 & \frac{A}{6} & 0 & 0\\
 & \frac{13A}{35}+\frac{6J_z}{5l^2} & \frac{11Al}{210}+\frac{J_z}{10l} & 0 & \frac{9A}{70}-\frac{6J_z}{5l^2} & -\frac{13Al}{420}+\frac{J_z}{10l}\\
 & & \frac{Al^2}{105}+\frac{2J_z}{15} & 0 & \frac{13Al}{420}-\frac{J_z}{10l} & -\frac{Al^2}{140}-\frac{J_z}{30}\\
 & & & \frac{A}{3} & 0 & 0\\
 & & & & \frac{13A}{35}+\frac{6J_z}{5l^2} & -\frac{11Al}{210}-\frac{J_z}{10l}\\
 & & & & & \frac{Al^2}{105}+\frac{2J_z}{15}
\end{bmatrix},
$$

The lumped mass matrix without rotations (diagonalised):

$$
m=\frac{\rho l A}{2}\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}.
$$

FOR 3D FRAMES

$$
M^e=\rho l\times \tag{6-24}
$$

$$
\begin{bmatrix}
\frac{A}{3} & 0 & 0 & 0 & 0 & 0 & \frac{A}{6} & 0 & 0 & 0 & 0 & 0\\
 & \frac{13A}{35}+\frac{6J_z}{5l^2} & 0 & 0 & 0 & \frac{11Al}{210}+\frac{J_z}{10l} & 0 & \frac{9A}{70}-\frac{6J_z}{5l^2} & 0 & 0 & 0 & -\frac{13Al}{420}+\frac{J_z}{10l}\\
 & & \frac{13A}{25}+\frac{6J_y}{5l^2} & 0 & -\frac{11Al}{210}-\frac{J_y}{10l} & 0 & 0 & 0 & \frac{9A}{70}-\frac{6J_y}{5l^2} & 0 & \frac{13Al}{420}-\frac{J_y}{10l} & 0\\
 & & & \frac{J_x}{3} & 0 & 0 & 0 & 0 & 0 & \frac{J_x}{6} & 0 & 0\\
 & & & & \frac{Al^2}{105}+\frac{2J_y}{15} & 0 & 0 & 0 & -\frac{13Al}{420}+\frac{J_y}{10l} & 0 & -\frac{Al^2}{140}-\frac{J_y}{30} & 0\\
 & & & & & \frac{Al^2}{105}+\frac{2J_z}{15} & 0 & \frac{13Al}{420}-\frac{2J_z}{10l} & 0 & 0 & 0 & -\frac{Al^2}{140}-\frac{J_z}{30}\\
 & & & & & & \frac{A}{3} & 0 & 0 & 0 & 0 & 0\\
 & & & & & & & \frac{13A}{35}+\frac{6J_z}{5l^2} & 0 & 0 & 0 & -\frac{11Al}{210}-\frac{J_z}{10l}\\
 & & & & & & & & \frac{13A}{35}+\frac{6J_y}{5l^2} & 0 & \frac{11Al}{210}+\frac{J_y}{10l} & 0\\
 & & & & & & & & & \frac{J_x}{3} & 0 & 0\\
 & & sym. & & & & & & & & \frac{Al^2}{105}+\frac{2J_y}{15} & 0\\
 & & & & & & & & & & & \frac{Al^2}{105}+\frac{2J_z}{15}
\end{bmatrix}
$$

FOR FINITE ELEMENTS

Numerical integration is used

$$
m=\int_V N^T\rho N dV
$$
, where *N – shape functions array*

MASS DIAGONALIZATION

1. Compute the consistent mass matrix $([M'_e])$ in the usual manner.
2. Compute:

$$S(i) = \sum_{j=1}^{n} M'_e(i,j) \quad \text{for } i = 1, n$$

n = number of degrees of freedom (DOFs) in the element

3. Set

$$M_e(i,j) = 0.0 \qquad \text{for } i \neq j$$

$$M_e(i,j) = S(i) \qquad \text{for } i = 1, n$$

For higher order elements the procedure suggested by Hinton, et al., is used. The steps are:

1. Compute the consistent mass matrix $([M'_e])$ in the usual manner.
2. Compute:

$$S = \sum_{i=1}^{n} \sum_{j=1}^{n} M'_e(i,j)$$

$$D = \sum_{i=1}^{n} M'_e(i,i)$$

3. Set:

$$M_e(i,j) = 0.0 \qquad \text{if } i \neq j$$

$$M_e(i,i) = \frac{S}{D} M'_e(i,i)$$

Note that this method ensures that:

1. The element mass is preserved
2. The element mass matrix is positive definite

Link to some reference discussing the diagonalisation of mass matrix:
http://www.colorado.edu/engineering/CAS/courses.d/MFEMD.d/MFEMD.AppD.d/MFEMD.AppD.pdf