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BUCKLING LENGTH FACTORS FOR WELDED LATTICE GIRDERS
with Hollow Section Braces and Chords

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INTRODUCTION

Eurocode 3 (EN 1993-1-1) presents buckling length factors for hollow section braces and chords in welded lattice steel girders: $K = 0.9$ for chord and $K = 0.75$ for brace members. In earlier versions of Eurocode 3, $K = 0.75$ for brace members was only allowed for $\beta \leq 0.6$, with β for rectangular or square hollow sections (RHS or SHS) being the ratio of the brace width b_1 to the chord width b_0 and β for circular hollow sections (CHS) being the ratio of the brace diameter d_1 to the chord diameter d_0. In the former Dutch code (NEN6770) the requirement for β was different: $\beta \geq 0.6$ for RHS or SHS and $\beta \geq 0.5$ for CHS. It is likely that also γ, the ratio of the width b_0 or diameter d_0 to two times the wall thickness t_0 of the chord, influences the buckling length factors. In general it can be stated that the stiffness of the connections of braces to chords will influence the buckling length factors of braces and chords in lattice girders. For these reasons it was decided to numerically investigate the influence of the parameters β and γ on connection stiffness and on the in-plane and out-of plane buckling length factors through linear buckling analyses (LBA) with Ansys 11.0.

1 LITERATURE

Apart from the rules in the codes as presented in the introduction, CIDECT [1] also provides brace buckling length factors, e.g. a formula for SHS braces connected to SHS chords:

$$K = 2.3\left(\frac{b_1^2}{L_1 b_0}\right)^{0.25} \quad \text{for } \beta < 0.6 \tag{1}$$

where L_1 is the brace length.

Similar formulae with different coefficients are provided in [1] for CHS braces connected to SHS and CHS chords. Earlier research on effective lengths of tubular truss members was presented in [2 to 4]. Later research [5] showed that $K = 0.75$ for out-of-plane brace buckling is on the conservative side for many cases but sometimes un-conservative. In a preliminary study [6], LBA of lattice girders modelled completely with shell elements, indicates a significant influence of β on the buckling length factors.

2 APPROACH

If a lattice girder is completely modelled with shell elements to perform LBA, the influence of connection stiffness on buckling length factors cannot be studied separately. Therefore, it was

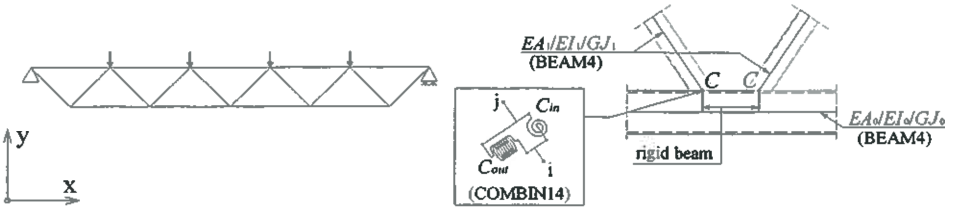
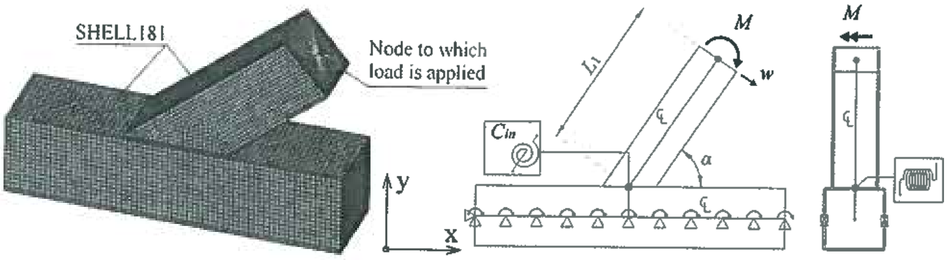


Fig. 1. FEM model with beam elements for braces and chords and spring elements for connections



FEM shell model
Hand calculation model

Fig. 2. In-plane connection stiffness calculation models

decided to model the braces and chords with beam elements (Beam4) and to model the hollow section connections with rotational springs (Combin14), see *Fig. 1*. The spring stiffnesses were determined separately by comparison of shell (Shell181) element model results with those of hand calculations. Lattice girder configurations were analysed subsequently in a parameter study.

3 CONNECTION STIFFNESS EVALUATION

Only gap connections have been considered as shown in *Fig. 1*. The connection stiffness is applied between two nodes i en j with the same coordinates located at the top of the chord where the chord wall intersects the brace system line (points C). The offset between the chord system line and point C is modelled by a rigid beam element fixed to the chord system line. Six stiffnesses can theoretically be attributed between nodes i and j, three translational and three rotational stiffnesses. The translational stiffnesses in x- and z-direction are assumed to be infinite. Preliminary calculations showed that the translational stiffness in the y-direction does not influence the LBA results. The rotational stiffness C_{in} for in-plane connection behaviour represents the rotational stiffness about the z-axis. The rotational stiffnesses about the x- and y-axes are captured by the out-of-plane rotational stiffness C_{out}. Thus, if the in-plane and out-of-plane rotational stiffnesses are known, the connection is sufficiently modelled for LBA.

These values are obtained through comparison of brace displacements obtained with a shell finite element model of the connection and obtained with a relatively simple hand calculation [7]. This is illustrated in *Fig. 2* for the in-plane rotational stiffness C_{in} where the chord is fully restrained. The displacement w_{fem} at the brace end due to a bending moment M can be calculated with the FEM shell model. Using the hand calculation model the displacement w_{hand} at brace end can be calculated, which comprises a contribution from the connection and a contribution from brace bending:

$$w_{hand} = \frac{ML_1}{C_{in}} + \frac{ML_1^2}{2EI_1} \tag{2}$$

where EI_1 is the brace bending stiffness.

By setting w_{hand} equal to w_{fem}, the in-plane rotational stiffness C_{in} can be evaluated. In the lattice girder of *Fig. 1*, two braces are present at a connection to the chord, except near the supports. Since the upper face of the chord will deform, the second brace will also show displacements due to the bending moment on the first brace. Therefore three load cases have been considered as shown in *Fig. 3*, each yielding its own value for C_{in}. For the out-of-plane rotational stiffness a similar

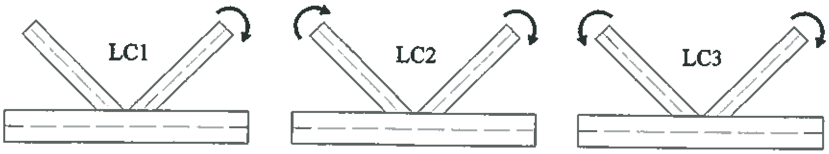
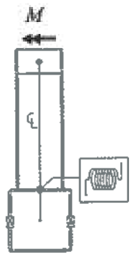


Fig. 3. In-plane load cases to determine connection stiffnesses



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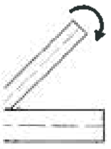


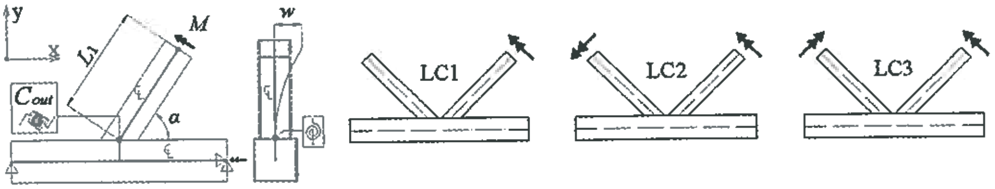


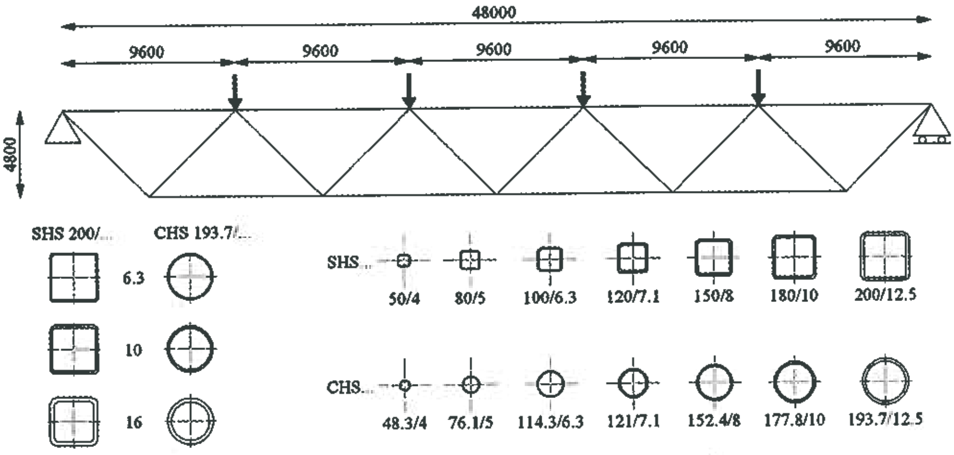
Fig. 4. Out-of-plane connection stiffness hand calculation model (left) and load cases (right)

procedure was followed using the hand calculation model of *Fig. 4* leading to more complex formulae [7]. Finally, a choice was made for the rotational stiffnesses to be used in the LBA. A comparison was made between LBA results using a full shell element model and a beam model with rotational springs, showing that for the in-plane rotational stiffness C_{in} the value obtained for load case 1 should be used, while for the out-of-plane rotational stiffness C_{out} the values obtained for the load cases 1, 2 and 3 should be averaged.

4 PARAMETER STUDY

A realistic welded Warren lattice girder was used to perform a parameter study varying the hollow section type of chords and braces and their dimensions, *Fig. 5*. All chords consist of the same cross-section; all braces have the same cross-section. The brace to chord combinations considered are:

- SHS braces to SHS chords with $\beta = 0.25, 0.40, 0.50, 0.60, 0.75, 0.90$, and 1.00 and $\gamma = 6.25$, 10.00 and 15.87 (*Table 1*; the combination $\beta = 1.00$ and $\gamma = 6.25$ leads to modelling errors);
- CHS braces to SHS chords with $\beta = 0.24, 0.38, 0.57, 0.61, 0.76, 0.89$, and 0.97 and $\gamma = 6.25$,



Chord cross-sections

Brace cross-sections

Fig. 5. Basic geometry of lattice girder used in parameter study and cross-sections used

Table 1. SHS brace to SHS chord combinations considered

Chord	Brace	β	γ	Chord	Brace	β	γ	Chord	Brace	β	γ
200/16	50/4	0.25	6.25	200/10	50/4	0.25	10.00	200/6.3	50/4	0.25	15.87
	80/5	0.40	6.25		80/5	0.40	10.00		80/5	0.40	15.87
	100/6.3	0.50	6.25		100/6.3	0.50	10.00		100/6.3	0.50	15.87
	120/7.1	0.60	6.25		120/7.1	0.60	10.00		120/7.1	0.60	15.87
	150/8	0.75	6.25		150/8	0.75	10.00		150/8	0.75	15.87
	180/10	0.90	6.25		180/10	0.90	10.00		180/10	0.90	15.87
	-	-	-		200/12.5	1.00	10.00		200/12.5	1.00	15.87

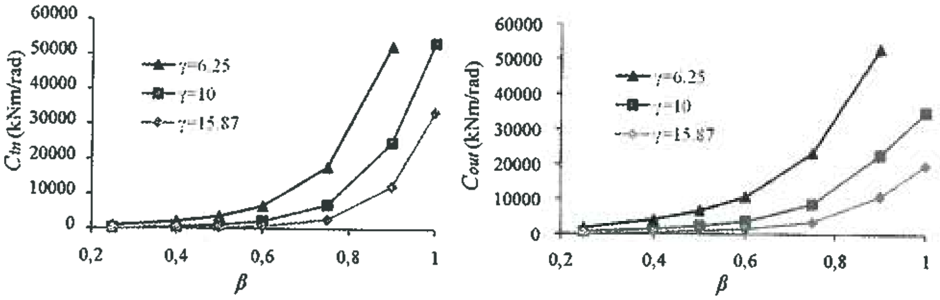


Fig. 6. In-plane stiffness (left) and out-of-plane stiffness (right) for SHS to SHS gap joints

10.00 and 15.87;

- CHS braces to CHS chords with $\beta = 0.25, 0.39, 0.59, 0.62, 0.79, 0.92$, and 1.00 and $\gamma = 6.05, 9.69$ and 15.37.

The connection gap g is taken equal to the minimum gap according to EN 1993-1-8, here twice the brace wall thickness t_1: $g = 2t_1$. This gap length was not varied since its influence was shown to be negligible for moderate gaps. The left support is a hinge and the right support is a roller. At every joint of the upper chord point loads are applied as well as restraints against out-of-plane translation.

5 INFLUENCE OF CONNECTION PARAMETERS ON ITS STIFFNESS

Rotational connection stiffnesses are determined as previously described. See [7] for all results. As a typical example of the rotational connection stiffness dependency on β and γ, *Fig. 6* shows the results for the in-plane stiffness C_{in} (for LC1, *Fig. 3*) and out-of-plane stiffness C_{out} (average value of those for LC1, LC2 and LC3, *Fig. 4*) of SHS braces connected to a SHS chord. As with increasing brace width β increases, the in-plane rotational stiffness C_{in} increases exponentially. So, if the width of the brace approaches the width of the chord, the stiffness increases. Force transfer from brace to chord is more direct with less influence of chord wall bending. As with decreasing chord wall thickness γ increases, the in-plane rotational stiffness C_{in} decreases. So, a decrease in chord wall thickness has a detrimental effect on the connection stiffness. The rotational connection stiffnesses C_{in} and C_{out} are implemented in the FEM model of *Fig. 1* to perform LBA for lattice girders.

6 BUCKLING LENGTH FACTORS

LBA were carried out. *Fig. 7* shows the four relevant buckling modes for SHS100/6.3 braces connected to a SHS200/6.3 chord. From these LBA, the bucking length factors of brace and chord members were derived. With the eigenvalue λ obtained from LBA and the normal force N in brace or chord known, the buckling length L_{cr} can be calculated using the Euler formula:

$$L_{cr} = \sqrt{\frac{\pi^2 EI}{\lambda N}} \tag{3}$$

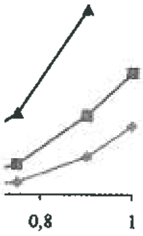
The buckling length factor K can now be obtained as follows:

$$K = L_{cr} / L_{sys} \tag{4}$$

in-plane brace buckling (side view) out-of-plane brace buckling (top view)

in-plane chord buckling (side view) out-of-plane chord buckling (top view)

Fig. 7. Buckling modes for lattice girder with SHS100/6.3 braces and SHS200/6.3 chord



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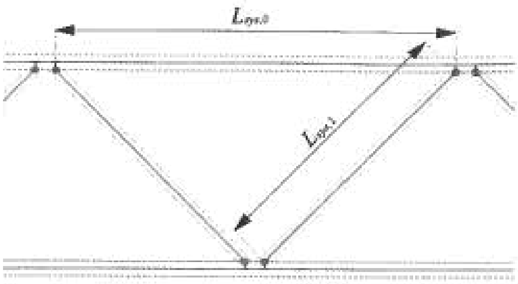

Fig. 8. System length definitions

where EI is the brace or chord bending stiffness,
L_{sys} is the system length as defined in *Fig. 8*.

The solid lines in *Fig. 9* show the resulting in-plane buckling length factors for SHS braces connected to SHS chords. Similar results were obtained for out-of-plane buckling length factors and for CHS braces connected to SHS and CHS chords.

In *Fig. 9a*, chord buckling length factors are shown in comparison to $K = 0.9$ as prescribed by Eurocode 3, the latter value being on the conservative side. With increasing β the value of K decreases while γ has a minor influence on K.

In *Fig. 9b*, brace buckling length factors are shown in comparison to $K = 0.75$ as prescribed by Eurocode 3 and to CIDECT *Eq. (1)*, the dashed line with circles indicating the β-range where *Eq. (1)* is not valid. The values for K based on Eurocode 3 and *Eq. (1)* are in many cases over-conservative but not always on the conservative side. With increasing β the value of K increases despite the connection stiffness increase (*Fig. 6*), due to the diminished support from the chords. With increasing γ and keeping β constant, also K increases due to the reduction of connection stiffness (*Fig. 6*).

7 FORMULAE FOR BUCKLING LENGTH FACTORS

New buckling length formulae were designed such that they give results which are maximum 5% on the un-conservative side when compared to the buckling length factors obtained with FEM. For brace members, the CIDECT formulae have been used as a base while introducing γ into the equations. The following formulae for buckling length factors are suggested:

- for chord buckling: $$K = A + B\beta \leq 0.9 \qquad (5)$$

where A and B are chord constants depending on the section type combination and on in-plane or out-of-plane buckling, see *Table 2*;

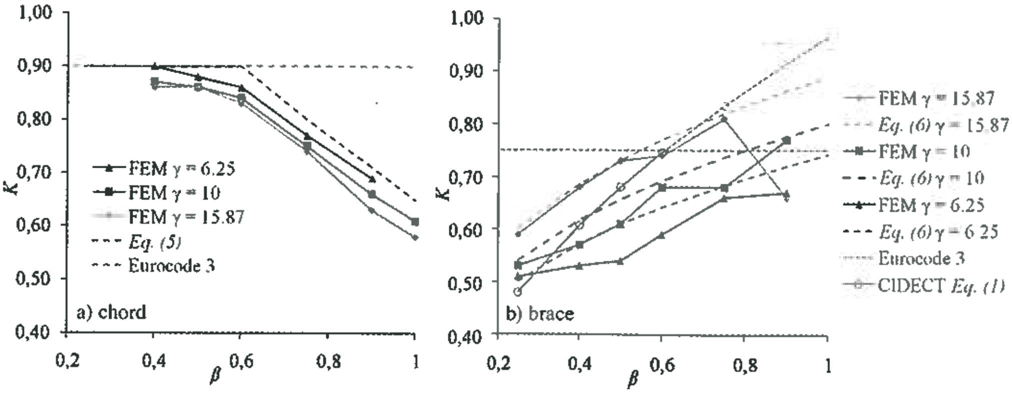

Fig. 9. In-plane buckling length factors for SHS braces to chords as function of β for different γ

- for brace buckling:

$$K = (C + D\gamma)\left(\beta \frac{b_1}{L_{sys,1}}\right)^E + F \qquad (6)$$

where C, D, E and F are brace constants depending on the section type combination and on in-plane or out-of-plane buckling, also see *Table 2*.

Table 2. Chord and brace constants in *Eq. (5)* and *(6)* for buckling length factors

Chord	Brace	In- / Out-of-plane	Chord constants		Brace constants			
			A	*B*	*C*	*D*	*E*	*F*
SHS	SHS	In	1.25	-0.60	1.05	0.025	0.14	0
SHS	SHS	Out	1.17	-0.45	3.0	1.2	1.0	0.55
SHS	CHS	In	1.17	-0.45	0.95	0.030	0.14	0
SHS	CHS	Out	1.11	-0.25	0	1.2	1.0	0.55
CHS	CHS	In	1.25	-0.60	2.5	1.0	1.0	0.50
CHS	CHS	Out	1.17	-0.45	5.0	1.0	1.0	0.55

For SHS braces to chords, a comparison of FEM results with those obtained with *Eq. (5)* and *(6)* is shown in *Fig. 9* as well. The dashed lines in *Fig. 9* indicate the results obtained from the equations.

8 DISCUSSION

The formulae proposed are valid for the truss and loading configuration as studied (*Fig. 5*). For this configuration, chord buckling is induced at mid span and brace buckling occurs in the compression brace nearest to the supports (*Fig. 7*). Chord and brace buckling are therefore uncoupled. However, if the cross-sections of chords and braces are optimised, chord and brace buckling may interfere, probably yielding greater values for the buckling length factors. This needs further research.

9 SUMMARY

Parameters β and γ are the salient parameters for buckling length factors of hollow section braces in welded lattice girders. For hollow section chords, only β is relevant. It was shown that existing formulae for buckling lengths factors are in many cases over-conservative and sometimes unconservative. Improved formulae for buckling length factors of hollow section brace and chord members in welded lattice girders with gap joints are proposed. For braces these new formulae are based on the existing CIDECT formulae, taking the effect of β and γ into account. Further research is needed for other truss and loading configurations than investigated here.

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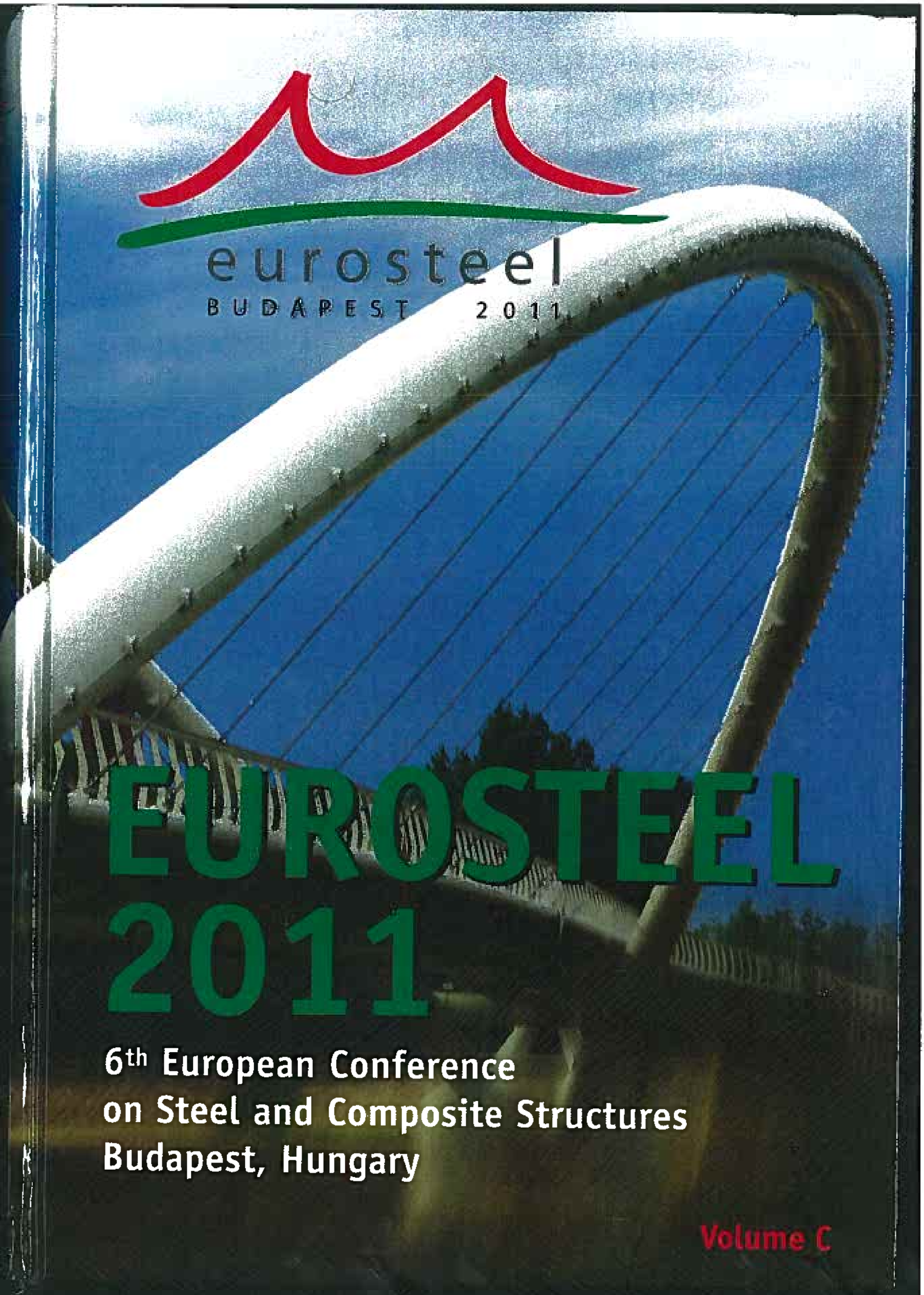

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