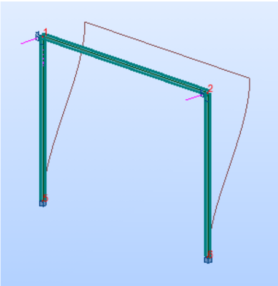
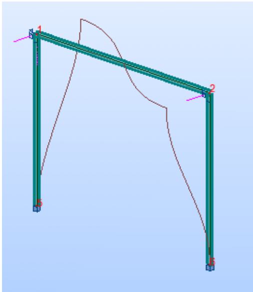
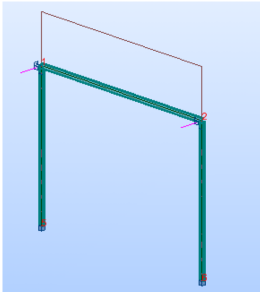
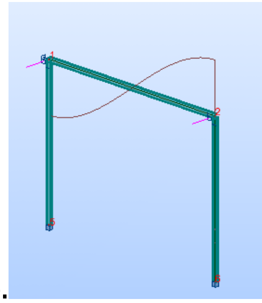
Modal masses for unsymmetrical modes

SIMPLE FRAME EXAMPLE

Having a 3D model (height equals 3,0 m, width - 4,0 m, all bars - IPE 100, lumped masses m=200 kg in both corners, totally blocked supports, blocked horizontal movements in-plane) we have 4 "dynamical" degree of freedom: { UY1, UZ1, UY2, UZ2 },

All (4) vibration mode shapes are presented on the figures below,

1.  2.  3.  4. 

and may be described by vectors Φ_i, i=1,...4:

Φ_1 = { 1; 0; 1; 0 };

Φ_2 = { 1; 0; -1; 0 };

Φ_3 = { 0; 1; 0; 1 };

Φ_4 = { 0; -1; 0; 1 }.

Mass matrix **M** is:

200	0	0	0
0	**200**	0	0
0	0	**200**	0
0	0	0	**200**

Substituting $m_i = \Phi_i \times M \times \Phi_i^T$ we have: m_2 = 400 kg, m_4 = 400 kg.

"Normalization" of Φ_i /divided them by sqrt(400)=20,0/ give us a set of V_i.

V_2 = { 0,05; 0; -0,05; 0 };
V_4 = { 0; -0,05; 0; 0,05 }.

MODAL PARTICIPATION FACTOR and EFFECTIVE MASSES in Y DIRECTION

Mode 1: MPFY2 = V_1 x **M** x $\{ 1; 0; 1; 0 \}^T$ = 20,0

Mode 2: MPFY2 = V_2 x **M** x $\{ 1; 0; 1; 0 \}^T$ = 0,0

Resulting EFFECTIVE MODAL MASSes /calculated as $(MPFYi)^2$ / (V_i x **M** x V_i^T)/ are:
400,0 kg for the mode 1, and ZERO for the mode 2 (in spite of NON-ZERO modal displacements)

Similarly, for Z directions:

MODAL PARTICIPATION FACTOR in Z DIRECTION (modes 3, 4)

MPFZ3 = V_3 x **M** x $\{ 0; 1; 0; 1 \}^T$ =20,0

MPFZ4 = V_4 x **M** x $\{ 0; 1; 0; 1 \}^T$ = 0

And EFFECTIVE MODAL MASSES are 400,0 and ZERO.

As one can see – unsymmetrical modes give no contribution in effective mass.

In other words, while moving a basement in some direction, un-symmetrical vibration (in a sense of this direction) will play no role (will not be observed).

The similar results are obtained in other programs e.g. SAP as shown below.